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KENYATTA UNIVERSITY

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN PETROLEUM ENGINEERING AND BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

EPL 510/EMM 514: CONTROL ENGINEERING II

1ST SEMESTER 2022/2023 TIME: 2 HOURS

INSTRUCTIONS:

1. This paper consists of **FIVE** questions.
2. Answer **QUESTION ONE** and **ANY OTHER TWO** questions.
3. All workings must be clearly shown.

QUESTION ONE (COMPULSORY) [30 MARKS]

a) For the PID controller shown in Figure Q1 a) below, determine the values of R_1 and R_2 such that the proportional gain constant is 8.5 and the derivative time constant is 5. Determine the resulting value of the integral constant. Take $C_1 = 200\ \mu F, C_2 = 50\ \mu F\ and\ R = 10\ k\Omega$

(6 mks)

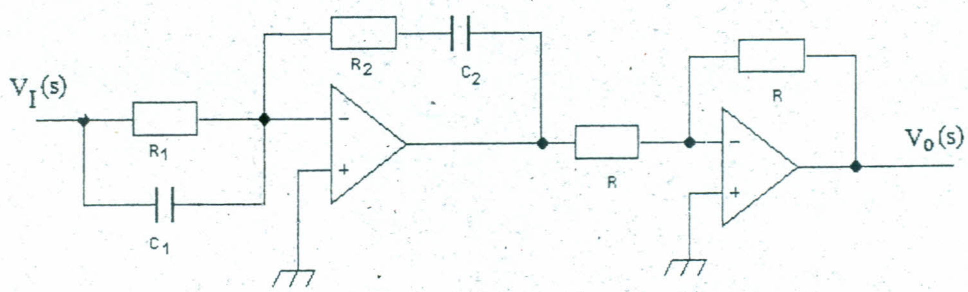

Figure Q1 a)

b) For the linear system defined by

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

i. Obtain the block diagram representation for the system

ii. Derive the expression for the transfer function of the system **(6 mks)**

c) Given the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u \quad \text{and } y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain the transfer function of the system **(4 mks)**

d) Obtain the mathematical model and the state space representation of the mechanical systems of Figure Q1 d) **(6 mks)**

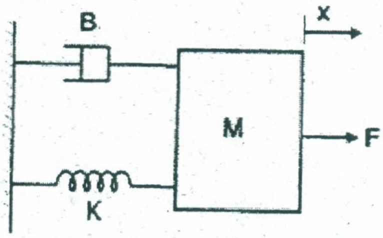

Figure Q1 d)

e) Obtain the describing functions for the nonlinearity shown in Figure Q1 e) **(6 mks)**

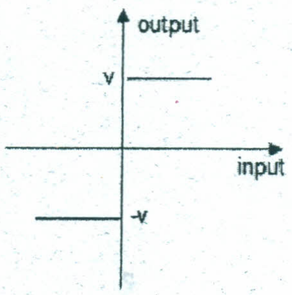

Figure Q1 e)

f) Explain the following optimal control problems with the aid of their mathematical expressions

i. Minimum time problem

ii. Minimum effort problem **(2 mks)**

QUESTION TWO [20 MARKS]

a) Discuss the Ultimate Cycle method of PID tuning with aid of suitable illustrations **(4 mks)**

b) Use the ultimate cycle method to determine the gains for a PID controller for the following plant given by

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$ **(8 mks)**

c) Explain the terms controllability and observability of control systems **(2 mks)**

d) For the linear system given by

$$\dot{x} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

Test for the controllability and observability of the system **(6 mks)**

QUESTION THREE [20 MARKS]

a) Given the control system of Figure Q2 a) where k *and* c are positive constants

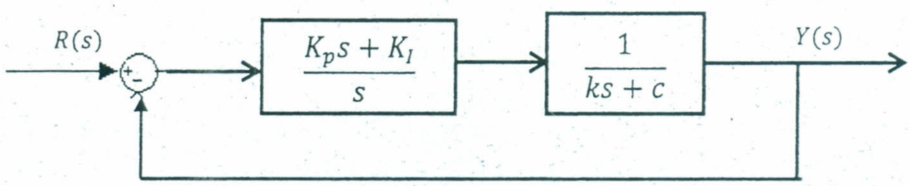

Figure Q2 a)

i. State the type of controller and order and type of system represented

ii. Obtain the closed loop transfer function for the system

iii. Determine the steady state value of the output for a unit step input

iv. Obtain the PI controller gain such that the damping factor ζ is 0.5 given that $k = 1$, $c = 1$ *and* $\tau = 0.2$ **(8 mks)**

b) Given the second order system

$$\dot{x}_1 = x_2(t)$$

$$\dot{x}_2 = u(t)$$

and performance index

$$J = \frac{1}{2}\int_0^2 u^2(t)dt$$

Determine the optimal control law and optimal state given the following boundary conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} x_1(2) \\ x_2(2) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ **(8 mks)**

c) With aid of suitable diagrams, explain the pole placement state space design approach **(4 mks)**

QUESTION FOUR [20 MARKS]

a) A system is described by the following state equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \qquad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Given that $u(t)$ is a unit step and $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ determine the state transition matrix and solve for $x(t)$ **(8 mks)**

b) Explain any **FOUR** properties of non-linear systems **(4 mks)**

c) A nonlinear system is described by

$$\ddot{y}(t) + \dot{y}(t) + y^2(t) + y(t) = 0$$

Let the state variable of the state equation be $x_1 = y, x_2 = \dot{y}$. Analyze the stability of the linearized model of the system about its equilibrium points. **(8 mks)**

QUESTION FIVE [20 MARKS]

a) Explain the term similarity transformation and state why it is useful in state space analysis **(2 mks)**

b) Given the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

Obtain the transformation matrix P from the Eigen vectors of A that would transform matrix A into diagonal canonical form and show that the characteristic equation of A does not change under the transformation **(10 mks)**

c) Consider the circuit of Figure Q5 c). Taking inductor current and capacitor voltage as the state variables, obtain the state space model of the circuit. **(8 mks)**

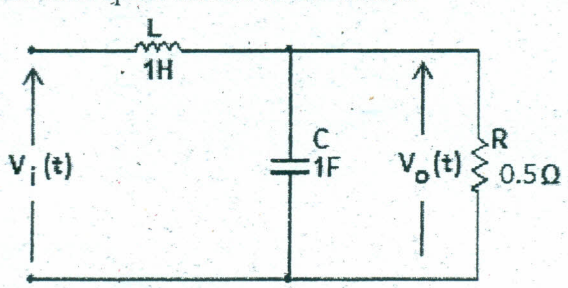

Figure Q5 c)

APPENDIX

A. Ziegler-Nichols Ultimate Cycle Table

Type of Controller	Optimum Gain		
	K_p	T_I	T_D
P	$0.5K_u$	−	−
PI	$0.45K_u$	$0.833p_u$	−
PD	$0.6K_u$	−	$0.125p_u$
PID	$0.6K_u$	$0.5p_u$	$0.125p$