KENYATTA UNIVERSITY

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE (COMPUTER SCIENCE)

SCO 805: NUMERICAL ANALYSIS IN COMPUTING

2ND SEMESTER 2023/2024 **TIME: 3 HOURS**

INSTRUCTIONS:

Answer Question **One** and any other **TWO** questions

QUESTION ONE (30 MARKS)

a) State one shortcoming of the Bisection method.

(1 mark)

b) Define a permutation matrix, hence give an example of a permutation matrix.

(2 mark)

c) State and briefly explain the pitfalls of the Newton-Raphson method.

(4 marks)

d) Using the first row evaluate

$$A = \begin{vmatrix} 1 & 4 & -3 \\ -5 & 2 & 6 \\ -1 & -4 & 2 \end{vmatrix}$$, hence determine the inverse of the matrix A.

(5 marks)

e) A function y of $x$is given by the following values:

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	0.0000	0.0499	0.0995	0.1483	0.1960	0.2423	0.2867

Apply Trapezium rule to approximate the value of $\int_0^{0.6} y dx$.

(3 marks)

f) Given the system

$$\begin{aligned} x_1 + x_2 \quad\quad + 3x_4 &= 8 \\ 2x_1 + x_2 - x_3 + x_4 &= 7 \\ 3x_1 - x_2 - x_3 + 2x_4 &= 14 \\ -x_1 + 2x_2 + 3x_3 - x_4 &= -7 \end{aligned}$$

Represent the linear system below as a matrix in the form $Ax = b$, hence determine the LU factorization for the Matrix A. (5 marks)

g) Find the eigenvalues and the associated eigenvectors for the matrix (5 marks)

$$A = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$$

h) Use Cramer's rule to solve the system

(5 marks)

$$\begin{aligned} x + y + z &= 4 \\ x - y \quad\quad &= 3 \\ x + 2y - z &= 0 \end{aligned}$$

QUESTION TWO (20 MARKS)

a) Estimate the value of the integral $\int_0^{0.4} \sqrt{1 - x^2}\, dx$ using Simpson's rule with 5 ordinates,

(5 marks)

b) Derive the Newton-Raphson method by considering a curve $y = f(x)$. (5 marks)

c) Find the minimum number of iterations required for converging to a root in the interval (0,1) for a given error $\epsilon = 10^{-6}$. (3 marks)

d) A d.c. circuit comprises three closed loops. Applying Kirchoff's law to the closed loops gives the following equations for to current flow in milliamperes:

$$\begin{aligned} 2l_1 + 3l_2 - 4l_3 &= 26 \\ l_1 - 5l_2 - 3l_3 &= -87 \\ -7l_1 + 2l_2 + 4l_3 &= 12 \end{aligned}$$

Use determinants to solve for l_1, l_2 and l_3. (7 marks)

QUESTION THREE (20 MARKS)

a) Perform five (5) iterations of the bisection method to obtain the smallest possible positive root
of the equations $f(x) = x^3 - 5x + 1 = 0$ (5 marks)

b) Determine the factorization in the form $A = (P^t L)U$ for the matrix

(5 marks)

$$A = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & 1 & -1 & 2 \\ -1 & -1 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{bmatrix}.$$

c) Solve the following differential equation using the 4th Order Runge-Kutta method.

(10 marks)

$\frac{dy}{dx} = t + y, y(0) = 1$ at $t = 0.2$ with $h = 0.1$

QUESTION FOUR (20 MARKS)

a) Represent the linear system below as an augmented matrix hence use Gaussian elimination to find its solution. (5 marks)

$$\begin{aligned} x_1 + x_3 - 2x_4 &= -3 \\ x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 4x_2 + x_3 - 3x_4 &= -2 \\ x_1 - 4x_2 - 7x_3 - x_4 &= -19 \end{aligned}$$

b) Find the eigenvalues and associated eigenvectors of the matrix (6 marks)

$$A = \begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$$

c) Use Euler's method to solve numerically the equation
$\frac{dy}{dx} = x + y,\ y(0) = 1$ for $x = [0, 1.0]$ using $h = 0.2$ (9 marks)