



KENYATTA UNIVERSITY
CITY CAMPUS
UNIVERSITY EXAMINATIONS 2016/2017
SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN
STATISTICS

SST 823: SURVIVAL ANALYSIS

DATE: Thursday, 11th May, 2017

TIME: 5.30 p.m. - 8.30 p.m.

INSTRUCTIONS: Answer question **ONE** and any other **TWO** questions.

Question 1(30 marks)

- a) Define the following terms as used in survival analysis.
- i) Survivor function (1 mark)
 - ii) Hazard function. (2 marks)

- b) The hazard function of failure time t is given by;

$$h(t) = \alpha\beta(\alpha t)^{\beta-1}(\alpha t)^{\beta}$$

Derive the survival function of T . (4 marks)

- c) If the p.d.f of the weibull distribution is given by ;

$$f(t) = \alpha\beta(\beta t)^{\alpha-1} \exp(-\beta t)^{\alpha}.$$

Determine the hazard function of T^{α} (6 marks)

- d) Let the possible values of T at which the probability mass function is non-zero be
 $u_0 \leq u_1 \leq \dots$

Show that the survivor function of T is given by $S_j = \prod_{j=0}^{t-1} (1 - h_j)$ where h_j denotes the hazard function. (6 marks)

- e) Suppose the life length of an electronic device is exponentially distributed. It is known that the reliability of the device for 100 hours period of operation is 0.90. How many hours of operation may be considered to achieve a reliability of 0.95. (4 marks)

- f) The following are the survival times in months of 48 patients suffering from myeloma disease;

13,52*,6,40,10,7*,66,10*,10,14,16,4
65,5,11*,10,15*,5,76*,56*,88,24,51,4
40*,8,18,5,16,50,40,1,36,5,10,91,
18*,18*,6,6,1,1,3,18,12*,12,17,3*

Using a class size of twelve, put this information in grouped form, hence compute the estimate of the survivor function

(7 marks)

Question Two(20 Marks)

Let there be two treatment groups denoted by 0 and 1, with c.d.f $G_0(t)$ and $G_1(t)$ respectively.

- a) Derive the generalized wilcoxon test statistic to test the hypothesis $H_0 : G_0(t) = G_1(t)$ against $H_0 : G_0(t) \neq G_1(t)$ (10 marks)
- b) Given the following failure times of two groups of patients receiving two malaria drugs.
Group A: 3, 5⁺, 7, 7⁺, 18.
Group B: 12, 19⁺, 20⁺, 21, 25⁺
 - i) Prepare an appropriate table, hence calculate the generalized wilcoxon test statistic. (8 marks)
 - ii) Assess the P-value of the computed value of the log rank statistic obtained in (a), hence test whether the two drugs are significantly different at 5% level of significance. (2 marks).

Question Three(20 marks)

- a) Suppose T is a discrete random variable with survivor function $S(t)$. Given the survivor function $S(t) = \prod_{u_j < t} (1 - h_j)$, Derive the
 - i) product limit limit estimator of the $S(t)$.
 - ii) Greenwood's formula for the variance of the product limit estimator. (14marks)
- b) The following are the times (in days) taken by 10 malaria patients to recover after taking some drugs.
2, 3, 4, 8*, 9, 10*, 12, 13, 18, 20*.
Calculate the product limit estimator using Efron's algorithm. (6 marks)

Question Four (20 marks).

- a) Distinguish between ;
 - i) Right censoring and left censoring.
 - ii) Type I and type II censoring. (4 marks)
- b) Let T be a non-negative discrete random variable with probability mass function $f(t, \theta)$ and hazard function $h(t, \theta)$. Let d_j and r_j be the number of individuals who fail and those who are at risk respectively at time u_j and h_j be the hazard function at u_j .
For the n ordered times $u_1 < u_2 < \dots < u_n$;
 - i) Obtain the likelihood function of T. (8 marks)
 - ii) Describe one method of making inference about θ . (3 marks)
- c) The following are the remission times of eleven cancer patients;
8, 13, 13⁺, 18, 21, 29⁺, 32, 34, 46⁺, 49, 163⁺
Fit an exponential distribution. (5 marks)