KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2014/2015

DIGITAL SCHOOL OF VIRTUAL AND OPEN LEARNING

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 104: CALCULUS I

DATE: Tuesday 28th April, 2015 **TIME: 2.00 p.m – 4.00 p.m**

INSTRUCTIONS:

Attempt question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

(a) Using the precise definition of limits show that

$\lim_{n\to\infty}\left\{\frac{(-1)^n}{n}\right\} \to 0$ where $n \in N$. Hence evaluate $N(\in)$ when

(i) $\varepsilon = 0.02$

(ii) $\varepsilon = 0.006$ **(4 marks)**

(b) Explaining every step evaluate the following limits

(i) $\mathrm{Lim}_{n\to\infty}\frac{3n^2-4n+9}{2n^2+5}$ **(2 marks)**

(ii) $\lim_{x\to 0}\frac{\sqrt[m]{1+ax}-\sqrt[n]{1+bx}}{x}$ **(2 marks)**

(iii) $\lim_{x\to\infty}\frac{(x-3)^{40}\,(5x+6)^{10}}{(3x^2-6)^{25}}$ **(2 marks)**

(iii) $\lim_{x\to 0}\frac{x^x\,\sin 2x}{3x}$ **(2 marks)**

(c) From the first principles find $\frac{dy}{dx}$ of the following functions;

(i) $f(x) = cosx$ **(3 marks)**

(ii) $f(x) = \sqrt{x+2}$ **(3 marks)**

(d) Let $\lim_{n\to\infty} X_n \to A$ and $\lim_{n\to\infty} y_n \to B$ Show that

(i) $\lim_{n\to\infty} x_n + y_n = A + B$ **(4 marks)**

(ii) $\lim_{n\to\infty} x_n\, y_n = A \cdot B$ **(4 marks)**

(e) Deduce the quotient rule of differentiation **(4 marks)**

QUESTION TWO (20 MARKS)

(a) Given $y = \frac{\cos x}{x}$, hence or otherwise prove that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$

(4 marks)

b) For what value of the constant B is the following function continuous for all $x \in \Re$

i) $F(x) = \begin{cases} X^2-3x+4 & \text{if } x>1 \\ Bx+5 & \text{if } x<1 \end{cases}$

ii) $F(x) = \begin{cases} 5X+B & \text{if } x>3 \\ x+B)^2 & \text{if } x<3 \end{cases}$

(6marks)

(c) let $f(x)=\sqrt{x}$, g(x)=4x-8 and h(x)=2x find f0g0h and hence find(f0g0h)$^{-1}$ (2marks)

(d) Determine whether $y = Ae^{ax} + Be^{-ax}$ is satisfied by $y'' - a^2y = 0$ **(3 marks)**

(e) Find the equation of the curve given the gradient is $4x - 2$ and $x -$ axis is the tangent.

(5 marks)

QUESTION THREE (20 MARKS)

(a) A particle moves along a straight line in such away that its distance from a fixed point on the line after t seconds is S meters, where $S = \frac{1}{6}t^3$. Find;

(i) Its velocity after 2 seconds and 3 seconds

(ii) Its acceleration after 1 seconds and 4 seconds **(5 marks)**

(b) A ball was thrown upwards with a velocity of 40 m/s. Find

(i) The acceleration, velocity and distance statements.

(ii) The maximum height the ball can attain (strictly use calculus techniques) **(6 marks)**

(c) The volume of a cylindrical tank is $32\pi mls$. Find the radius of the tank if the area have to be least. **(5 marks)**

(d) A 525 meter wire mesh was provided to fence a rectangular (like) plot. Find the maximum area it can enclose by mesh without any loss. **(4 marks)**

QUESTION FOUR (20 MARKS)

(a) Use the first principle to find $\frac{dy}{dx}$ of the following functions

(i) $y = lnx$ **(4 marks)**

(ii) $y = \cos x$ **(4 marks)**

(b) Given $y = 2x^3 - 15x^2 + 24x + 19$ find the stationary points. **(4 marks)**

(c) Differentiate the following functions.

(i) $y = \text{Sin}^3 2x$ **(2 marks)**

(ii) $y = \frac{Tan^2 e^x\{\cos x\}}{x^2}$ **(2 marks)**

(d) Evaluate (i) $\text{Lim}_{x \to \infty} \left\{\frac{(3x+1)}{3x-2}\right\}^{2x}$ **(2 marks)**

(ii) $\text{Lim}_{x \to 0} \frac{1-\cos 3x}{x^2}$ **(2 marks)**

QUESTION FIVE (20 MARKS)

(a) Find y' and y'' given $y = \dfrac{e^{ax} + e^{-ax}}{e^{ax} - e^{-ax}}$ **(6 marks)**

(b) Find y'' given $y = e^{-2x} \sin 4x$ **(5 marks)**

(c) Find y' and y'' given $x^4 + xy^3 + y^3 = 32$ at the point $(1, 1)$ **(5 marks)**

(d) Show that $\lim_{x \to 2} x^2 + 2x + 8 = 16$ **(4 marks)**