KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE (STATISTICS)

SST 807: STOCHASTIC PROCESSES I

DATE: Tuesday, 6th February 2018 **TIME: 5.30 p.m. - 8.30 p.m.**

INSTRUCTIONS:

Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS)

a) Let $a_0, a_1, a_2, \ldots, a_n, \ldots$ be a sequence of numbers satisfying the recurrence relation $a_n + a_{n-1} - 16\, a_{n-2} + 20\, a_{n-3} = 0 \quad for \quad n \geq 3$ subject to the initial values $a_0 = 0, a_1 = 1 \quad and\ a_2 = -1$. Find the general formula for the terms of the sequence. (6 marks)

b) Given that the bivariate generating function of $Y_m\ and\ Y_n\ is\ G_m\left[s_1\, G_{n-m}(s_2)\right]$, show that $E(Y_m\, Y_n) = \mu^{n-m}\, E(Y_m^2) \quad if \quad n > m \quad and \quad \mu = E(Y)$. (6 marks)

c) Show that for the linear growth process the first moment $M_1(t)$ satisfies the differential equation $M_1'(t) = (\lambda - \mu)\, M(t)$ (6 marks)

d) Let $G(s) = a\, s^2 + bs + c$, where a, b and c are positive integers and $G(1) = 1$. Assume that the probability of extinction is d ($0<d<1$). Show that $d = \frac{c}{a}$. (6 marks)

e) For a branching chain, calculate the probability of extinction for

$$p_0 = \frac{1}{6}, p_1 = \frac{1}{2}, p_2 = \frac{1}{3}$$

(6 marks)

INVOLVEMENT IN ANY EXAMINATION IRREGULARITY SHALL LEAD TO DISCONTINUATION

QUESTION TWO- (20 MARKS)

a) Given that the p.g.f of the nth generation is $G_n(s) = G_{n-1}[G(s)]$, μ is the expected number of descendants in the first generation and σ^2 is the variance of the first generation. Show that the mean and variance of the nth generation are μ^n and $\sigma^2 \mu^{n-1} \frac{(1-\mu^n)}{(1-\mu)}$, $\mu \neq 1$ respectively. (10 marks).

b) In a certain biological system each individual during its lifetime gives rise to available number k of new individuals who in turn produce further individuals during their lifetimes. The probability distribution of the variate k is $p_0 = \frac{1}{4}, p_1 = \frac{1}{2}, p_2 = \frac{3}{16}, p_3 = \frac{1}{16}$ and $p_k = 0$ for all $k \geq 4$.

Assuming that the system starts with one individual and the number of individuals born to any one individual is independent of the number born to another, find

i) The expected number of individuals in the population up to and including the nth generation, (3 marks)

ii) The variance of the number of individuals in the nth generation, (4 marks)

iii) The probability that the family tree will have to become extinct ultimately (3 marks)

QUESTION THREE (20 MARKS)

a) Write down the difference – differential equation for the linear growth process with immigration, where $\lambda_n = n\lambda + a$, $\mu_n = n\mu$ (2 marks)

b) Show that M(t) = E{X(t)} satisfies the differential equation $M'(t) = (\lambda - \mu) M(t) + a$ (5 marks)

c) Hence show that with initial conditions M(0) = i, if X(0) = i, then

$$M(t) = \frac{a}{\lambda - \mu}\{e^{(\lambda-\mu)t} - 1\} + i\, e^{(\lambda-\mu)t}, \lambda \neq \mu$$ (8 marks)

d) Taking limit as $\lambda \to \mu$, show that $M(t) = at + i$ for $\lambda = \mu$ (3 marks)

e) What is the limit of M(t) as $t \to \infty$ for $\lambda < \mu$? (2 marks).

QUESTION FOUR (20 MARKS)

a) Let $X(t) = B_0 + B_1\, t + B_2\, t^2$, where B_i, $i = 0,1,2$ are uncorrelated random variables with mean 0 and variance 1. Find the mean value function and the covariance function of{X(t)}. (5 marks)

b) The Polya process is a non- stationary pure birth process with λ_n depending on time,

$$\lambda_n = \lambda\left(\frac{1+an}{1+\lambda at}\right).$$

i) Show that the solution with initial conditions $p_n(0) = \begin{cases} 1, & n = 0 \\ o & otherwise \end{cases}$ is

$$p_n(t) = \frac{(1+a)(1+2a)\ldots\{1+(n-1)a\}}{n!}(\lambda at)^n(1+\lambda at)^{-\frac{1}{a}-n}$$ (12 marks)

ii) Show from the differential equation that the mean and variance are λt *and* $\lambda(1+\lambda at)$ (3 marks).