KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF SCIENCE

SPH 200: MECHANICS II

DATE: Friday 2nd December, 2011 **TIME:** 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS:

Answer question **one** and any other **two** questions. Question one carries 30 marks and other questions 20 marks each.

Question 1

a) i) Explain what a rigid body is? (1 mk)

ii) Explain what you understand by moment of inertia. (1 mk)

b) State one factor that determine the moment of inertia of a body. (1 mk)

c) i) State the principle of conservation of mechanical energy. (1 mk)

ii) Explain the variation of the kinetic and potential energies with displacement of a simple harmonic oscillator. Illustrate your answer with suitable graphs. (2 mks)

d) i) Give one example of a conservative force and state one property of a conservative force. (2 mks)

ii) Investigate whether the force $\mathbf{F}=(x^2+y^2)\,\mathbf{i}+2xy\,\mathbf{j}$ is a conservative force. (2 mks)

e) A particle at a point $(0,1,2)$ is acted upon by a force $\mathbf{F}=3\,\mathbf{i}+2xy\,\mathbf{j}+xz^2\,\mathbf{k}$. Calculate the work done in moving it to a point $(4,4,4)$. (Assume the axis are labeled in S I units) (3 mks)

f) If force $\mathbf{F}=\mathbf{i}-3\mathbf{j}$ is applied to a point whose position vector is $\mathbf{r}=3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, determine;

i) $\Gamma=\mathbf{r}\times\mathbf{F}$. (2 mks)

ii) find the angle between the two given vectors. (2 mks)

iii) find the unit vector $\hat{n}$ parallel to vector Γ. (1 mk)

g) A wheel having a diameter of $0.2m$ starts from rest and accelerates with constant angular acceleration to an angular velocity of $15rev.s^{-1}$ in $5s$. Find the angular velocity ω and the angular displacement at the end $5s$ of a point originally at the top of the wheel. (4 mks)

h)

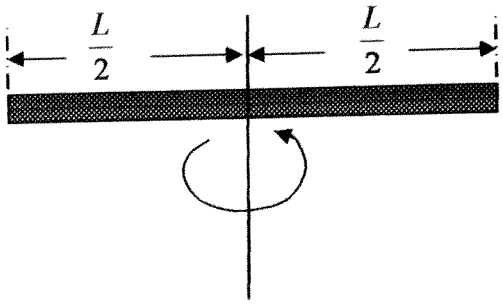

Figure 1

i) Figure 1 shows a **thin uniform** horizontal rod of length L, being rotated about a vertical axis passing through its geometrical center. Taking M to represent the mass of the rod, prove that the moment of inertia, I, of the system is given by $I = \frac{1}{12}ML^2$ (4 mks)

(ii) State the parallel axis theorem. (1 mk)

iii) If $L = 0.6m$ and $M = 2.0Kg$, use the parallel axis theorem to calculate the moment of inertia, I, of figure 1 when the axis of rotation is taken to be at one end. (2 mks)

I) State the necessary and sufficient condition for a body that is executing simple harmonic motion. (1 mk)

Question 2

a) Figure 2 shows two small bodies, which can be considered as particles, each of mass $200g$ whose centers are joined together by means of a horizontal light rod of length $0.8m$. If the system is rotated about the vertical axis passing through one-quarter of its length from one end, calculate the moment of inertia of the system. (3 mks)

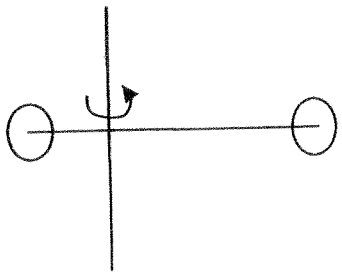

Figure 2

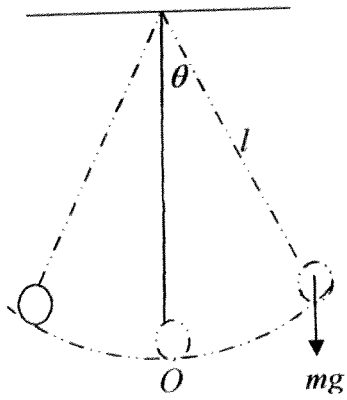

Figure 3

b) i) Figure 3 shows a light thread of length l clamped at one end while the other end is connected to a bob of mass m. Prove that when the body is displaced from its equilibrium position O through a small angle θ, it undergoes simple harmonic motion. (4 mks)

ii) Derive an expression for;

I) Its frequency (2 mks)

II) Its periodic time. (2 mks)

III) State two disadvantages of a simple pendulum. (2 mks)

c) For a boy to strike a bird using a catapult, he applied a force of $30N$ so as to stretch the catapult $10cm$. Calculate:

i) The elastic potential energy stored in the stretched catapult. (2 mks)

ii) If the stretched catapult has a small stone of mass $10g$, calculate the velocity at which the stone leaves the catapult. (2 mks)

d) Figure 4 shows a flywheel mounted on a horizontal axle which has a diameter of $0.12m$. A constant force of $50N$ is applied tangentially to the axle as shown in the figure below. If the moment of inertia the system (flywheel and axle) is $4kg.m^2$, calculate the angular acceleration. (3 mks)

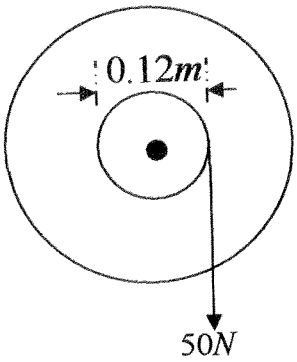

Figure 4

Question 3

a)

i) Explain what you understand by damped oscillation. (1 mk)

ii) Sketch on different axes curves for amplitudes versus time over three cycles for a damped harmonic oscillator and for un-damped simple harmonic oscillator. (2 mks)

b) The potential energy corresponding to a certain two-dimensional force field is given by $U(x,y)=\frac{1}{2}k\left(x^2+y^2\right)$. Derive F_x and F_y and describe the vector force at each point in terms of its Cartesian co-ordinates x and y. (4 mks)

c) A particle executes linear harmonic motion about the point $x=0;\ t=0$, it has displacement $x=0.37cm$ and zero velocity. If the frequency of the motion is $0.25s^{-1}$, determine

i) The period

ii) The amplitude,

iii) Its frequency (5 mks)

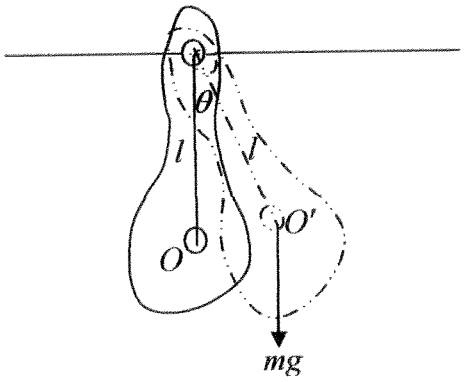

Figure 5

d) Figure 5 shows a compound pendulum of mass m and length l being suspended from a horizontal axis.

i) What is a compound pendulum? (1 mk)

ii) Explain what you understand by the term point of suspension as applied in compound pendulum. (1 mk)

iii)What do you understand by radius of gyration? (1 mk)

iv) Given that the moment of inertia of the above compound pendulum is I while its moment of inertia about its center of gravity is given by $I_o=mk^2$,where m and k have their

usual meaning, prove that when the above compound pendulum is displaced through a small angle θ it executes SHM and its periodic time is $T = 2\pi\sqrt{\frac{m(k^2+l^2)}{mgl}}$ (4 mks)

Question 4

a) A constant force of $30N$ is applied tangentially to the rim of a wheel mounted on a fixed axle and which is initially at rest. The wheel has a moment of inertia of $0.20kg.m^2$ and a radius of $15cm$.

i) What is the torque acting on the wheel? (2 mks)

ii) Find the work done on the wheel in 10 revolutions. (3 mks)

iii) Assuming no work done against friction, use energy considerations to find the angular velocity of the wheel after 10 revolutions. (3 mks)

b)

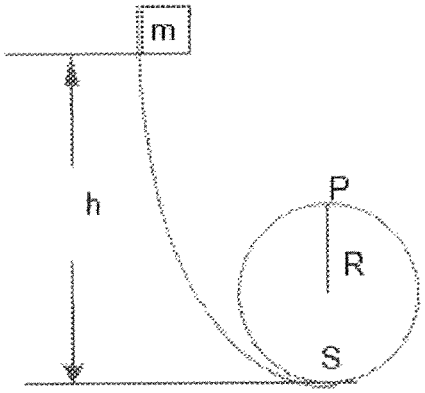

Figure 6

Figure 6 shows a block of mass m sliding along a frictionless loop-the-loop track from height h. Write down;

i) An expression of its velocity at lowest point S. (2 mks)

ii) An expression of velocity at point P which is at the top of the circular path. (2 mks)

iii) The reaction at point P. (1 mk)

c) Figure 7 shows a horizontal **thin disc** of radius, r, being rotated about a vertical axis. If its mass is M, derive an expression of its moment of inertia, I. (3mks)

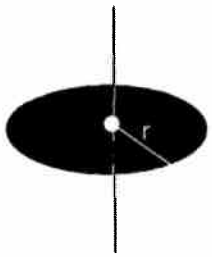

Figure 7

d) Solve the differential equation $\frac{d^2x}{dt^2} + \omega^2 x = 0$ to obtain the expression for the displacement x of a particle executing S.H.M. (4 mks)

Question 5

a) A hoop of radius $0.5m$ and mass $16kg$ is rolling along a horizontal floor, such that its centre of mass has a velocity of $0.25ms^{-1}$. What work will have to be done to stop it? (4 mks)

b) Figure 8 shows a wheel of radius $6cm$ mounted so as to rotate about a horizontal axis through its centre. A string of negligible mass, wrapped round its circumference, carries a mass of $200g$ attached to its free end. When let fall, the mass descends through $100cm$ in the first $5\ s$. Calculate the angular acceleration of the wheel and its moment of inertia. (6 mks)

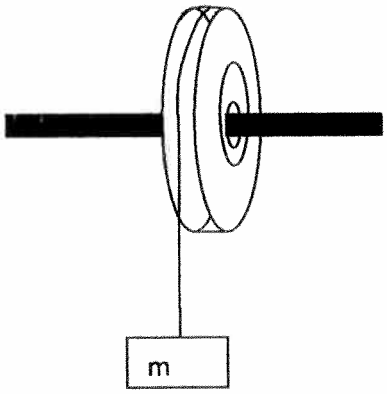

Figure 8

c) Show that if a stick of length l is mounted so as to rotate about a horizontal axis perpendicular to the stick and at a distance d from its center of mass, the period has the **minimum** value when $d = \frac{l}{\sqrt{12}} = 0.289l$.[Hint: this is about S.H.M.] (5 mks)

d) i) State the law of conservation of angular momentum. (1 mk)

ii)A particle of mass $5g$ moves in the spaces a long a curved path and its position at time t is defined by

$\mathbf{r} = (2t^3 + t)\mathbf{i} + (3t^4 - t^4 + 8)\mathbf{j} - 12t^2\mathbf{k}$.

Find;

I) the expression for the linear velocity of the particle. (2 mks)

II) The momentum of the particle at time $t = 1$ sec. (2 mks)