KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SPH 300: WAVE THEORY

DATE: Thursday, 8th December, 2011 TIME: 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS:

Answer question **one** and any other **two**. Question one carries 30 marks while each of the others carries 20 marks. Credit will be awarded for clear explanations and illustrations.

Useful information

1. Acceleration due to gravity	$9.8\ ms^{-2}$
2. The speed of electromagnetic waves in free space	$3.0 \times 10^{8}\ ms^{-1}$
3. The speed of sound in air	$3.3 \times 10^{2}\ ms^{-1}$
4. Mass of an electron	$9.11 \times 10^{-31} kg$
5. Permittivity of free space	$8.85 \times 10^{12}\ N^{-1}C^{2}m^{-2}$

$$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi$$

$$\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta-\alpha}{2}\right)$$

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots\ldots\ldots\ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \ldots\ldots\ldots\ldots$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$1 - \cos2\theta = 2\sin^2\theta$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \text{-----------}$$

$$\sin(\theta + \phi) = sn\theta\cos\phi + \cos\theta\sin\phi \ldots\ldots\ldots\ldots\text{(i)}$$

$$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$

Question one

(a) An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 36 cm. Calculate the (i) period, (ii) frequency and (c) amplitude of the motion. [5]

(b) A 1.0 μf capacitor is charged to 50 volts. The charging battery is then disconnected and a 10 mH coil is connected across the capacitor so that LC oscillations occur. What is the maximum current in the coil? Assume that the circuit contains no resistance. The following information may be useful; Energy stored in a capacitor and inductor:

$$U_E = \frac{1}{2}\frac{q_m^2}{C} \quad \text{and} \quad U_B = \frac{1}{2}Li^2$$, respectively. [5]

(c) A free, damped harmonic oscillator, consisting of a mass, m = 150 grams, which is moving in a viscous fluid of damping coefficient b, and attached to a spring of spring constant k = 0.9 Nm^{-1}, is observed as it performs damped oscillatory motion. Its average energy decays to 1/e of its initial value in 6 seconds. Determine the quality factor of the oscillator. [5]

(d) Fig. 1. shows two identical pendulums, A and B, which are connected together with a spring of length l and drawn aside so that the spring is stretched by an arbitrary amount (x_A–x_B). The respective equations of motion for the pendulums can be written in the form,

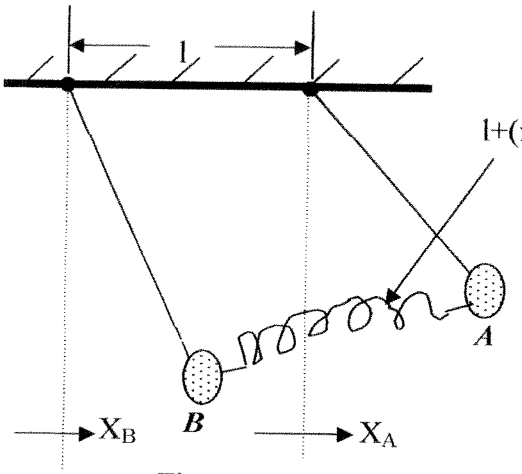

Fig. 1

$$x_A = \tfrac{1}{2} C\cos\omega_0 t + \tfrac{1}{2} D\cos\omega' t$$

and

$$x_B = \tfrac{1}{2} C\cos\omega_0 t - \tfrac{1}{2} D\cos\omega' t,$$

where C and D are constants while ω_0 and ω' are the corresponding normal frequencies. Apply the above equations to the analysis of the motions of two coupled pendulums in which one pendulum is initially displaced. Proceed as follows; (i) State the initial conditions of the motion in Fig. 1. (ii) Determine the displacements for the two pendulums. (iii) Briefly explain the relationship between the two amplitudes. [5]

(e) The frequency of vibration for the transverse vibrations of particles in a stretched string (many coupled oscillators) is given by,

$$\omega_n = 2\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right]$$

By an appropriate modification of this equation, determine an expression of the linear frequency (ν_n) in terms of the tension (T) in the string and the mass density (μ) of the string. [5]

(f) A very long uniform string of mass density 0.1 kgm^{-1} is stretched with a force of 50 N. A transverse traveling wave is set up in it. The wave equation is

$$y(x, t) = 0.02\sin 0.21(x - vt),$$

where x and y are in centimetres and t is in seconds. Find the (i)) wavelength, (ii) wave-number, (iii) frequency and (iv) period of the wave. [5]

Question two

(a) Electrons in an oscilloscope are deflected by two mutually perpendicular electric fields in such a manner that the displacement at any time is given by the equations,

$$x = A_1 \sin(\omega t + \delta) \text{ and } y = A_2 \sin \omega t.$$

If the phase difference between the vibrations is zero, show that the resultant path of the motion due to the superposition is a straight line. [4]

(b) i) Write the differential equation for a simple pendulum oscillating with SHM.
ii) Show that the period of oscillation for the simple pendulum is $2\pi(l/g)^{1/2}$.
iii) Determine the period of oscillation of a pendulum of length 1.5 m. [2,5,3]

(c) Evaluate the maximum tension in the string of a pendulum, if the mass and length of the bob are 0.1 kg and 1.5 m respectively, and oscillates with a maximum displacement of 0.15 m. [6]

Question three

(a) A block attached to a spring undergoes simple harmonic motion on a horizontal frictionless surface. Its total energy is 100 J. Calculate the kinetic energy when the displacement is half the amplitude. [5]

(b) Describe how the diffraction phenomenon occurs in crystals. [5]

(c) A single-loop circuit containing an inductance L, a capacitance C, and a resistance R, are connected to a sinusoidal oscillator of e.m.f. 1V, peak value. The inductivity is 0.1 mH, capacitance is 100 pF and the resistance is 200 Ω.

i) If the voltages in the circuit are related by the equation, $V_R + V_C + V_L = V_0\cos\omega t$, write the corresponding differential equation for the oscillations of the charge in the circuit.

ii) By comparing with the standard harmonic oscillator, evaluate the angular frequency of vibration of this electrical system. [2,8]

Question four

(a) Analyze the effect of subjecting the motion of a damped harmonic oscillator to an oscillatory, time-varying external driving force ($F = Fo\cos\omega t$) as follows. (i) Write the differential equation for the motion, (ii) use the trial solution $z = Ae^{j(\omega t-\delta)}$ to determine the amplitude (A) and phase (δ) of the motion and (iii) sketch the variation of amplitude and phase with driving frequency. [2,8,6]

(b) A 20 gram oscillator is driven by a sinusoidal force $F = F_o\cos\omega t$, where the constant force amplitude, $F_o = 2$ N and the frequency, $\omega = 30\ \text{sec}^{-1}$. If the dissipation constant $b = 4\ \text{Nm}^{-1}\text{s}$ and the elastic constant $k = 80\ \text{Nm}^{-1}$, evaluate the amplitude of the steady-state response described by the function, $x = A\cos(\omega t - \delta)$. [4]

Question five

(a) Newton's law for the transverse displacements of the p^{th} particle in a line of N identical coupled oscillators, in a flexible elastic string (Fig. 2) can be written as,

$$\frac{d^2 y_p}{dt^2} + 2\omega_0^2 y_p - \omega_0^2\left(y_{p+1} + y_{p-1}\right) = 0.$$

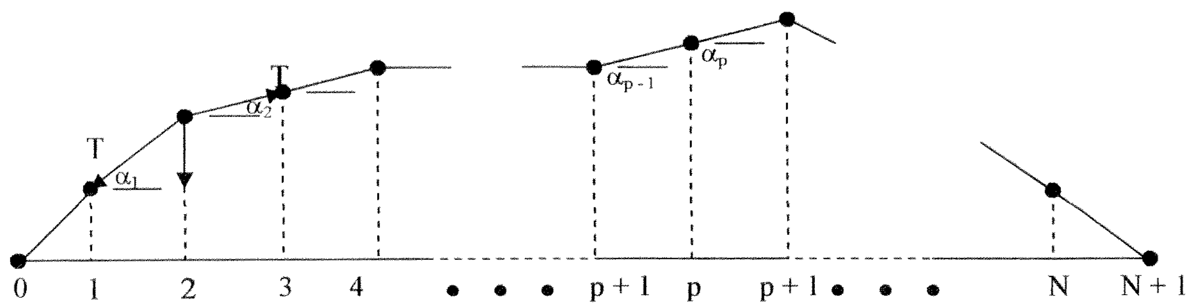

Fig. 2. Force diagram for transversely displaced masses on a long string

On the basis of the sinusoidal solution to this differential equation, show that the expression for the frequencies (ω_n) of the normal modes of many coupled oscillators is given by,

$$\omega_n = 2\omega_0 \sin\left[\frac{n\pi}{2(N+1)}\right].$$

Use the trial solutions,

$y_p = A_p\cos\omega t$; $A_p = C\sin p\theta$, $p = 1, 2, 3,$ ----------N [10]

(b) Expressed in mathematical form, Fourier's theorem asserts that, for a periodic function of fundamental frequency f_0,

$$g(t) = \sum_{n=0}^{\infty} A_n \cos(2\pi n f_0 t + \phi_n)$$

where the suffixes on the A_s and the ϕ_s denote that these latter belong to a particular n. With the aid of this expression, demonstrate how a given periodic function can be Fourier-analyzed, i.e., obtain the amplitude B_n associated with any given value of n in the harmonic analysis of a function y(x) using the following displacement equation,

$$y_n(x,\ t) = A_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t - \delta_n).$$ [10]