KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2011/2012
FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION (SCIENCE)

SPH 301: QUANTUM MECHANICS I

DATE: Monday, 5th December, 2011 TIME: 2.00 p.m. – 4.00 p.m.

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions. Question **ONE** carries 30 marks, other questions carry 20 marks each.

Q1.

a) i). For a particle confined in a given space, its wave function must be normalizable in that space. Explain this statement? (2 marks)

ii) An electron is moving in a one-dimensional box of infinite height whose width is b, find the expression for B if the wave function of the electron is defined by the expression

$$\phi(x) = B \sin \frac{n\pi}{b} x$$ (4 marks)

b) Define the expression $|\phi(x)|^2$ for a wave function $\phi(x)$ and explain the implication of it being infinite. (3 marks)

ii) For the electron in question 1(b), find the value of $|\phi(x)|^2$ at the centre of the box whose width is 25×10^{-10} m, when the electron is at its minimum energy. (3 marks)

iii) What is the probability that it would be found in a space of length 5×10^{-10} m at the centre of that box? (2 marks)

c). What is degeneracy of states? Explain the features of atomic spectra that reveal the existence of the degeneracy of states. (4 marks)

b) State and explain one of the quantities that would be altered by altering the size of the region of confinement of a particle in a force free region (2 marks)

ii) Describe how this information would be useful for the development of materials for computers. (3 marks)

c) i). What is an operator ? Give an example (2 marks)

ii) Starting from a suitable one-dimensional time independent Schroedinger equation, show that the momentum and the Kinetic energy operators are

1) $p = i\hbar \dfrac{\partial}{\partial x}$ and (3 marks)

2) $\hat{E} = -\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$ (2 marks)

Q2.

a). i) Explain the concept of "tunneling through" of an electron, and state conditions under which it may occur. (2 marks)

ii) Draw a labeled sketch of a one-dimensional periodic potential in a crystal, state where the potential would be minimum and maximum (3 marks)

iii) Using a diagram, describe an experiment in which the principle of tunneling can be illustrated using a beam of light (3 marks)

b) Write the expression for the three dimensional time independent Schroedinger equation for a particle in a force-free field and show clearly how an expression for its solution can be obtained. (6marks)

ii). Write the energy expression associated with a square well potential of width b and infinite height. Determine the number of degenerate states which would result for the second energy state (3 marks)

iii). Calculate the energy difference between the minimum and the second energy sates for electron in the potential well of question b(ii) for $b = 2 \times 10^{-8}$m (3 marks)

Q3.

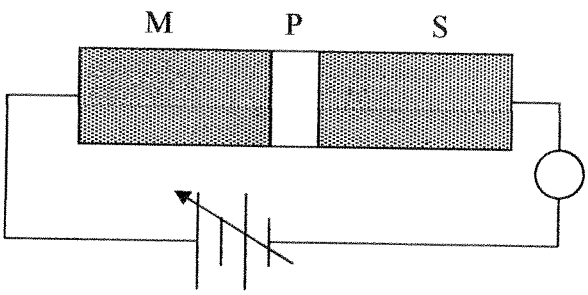
a)i) Write the expression for the penetration depth Δx of a particle into a potential barrier of potential V. (2 marks)

ii). If electron exhibits a penetration depth of 10^{-12} m through a barrier, determine the difference between its energy and the barrier potential (2 marks)

iii) Explain the difference between an infinite potential well and a finite potential well with respect to their effect on a confined particle. (3 marks)

b) i) Explain how temperature influences the minimum possible energy of the classical and quantum particles (3 marks)

ii) Sketch and label the ϕ (x) diagrams for the first 3 bound states of a particle in a **finite** and in an **infinite** potential wells (4 marks) iii). In an experiment set up to establish the tunneling nature of electron current, an insulator P is placed between two metal conductors M and S as shown.



Describe how you would use it to demonstrate and explain the tunneling nature of the current. (6 marks)

Q4

a). i). What are orthogonal wave functions? (1 mark)

ii). Show that for two wave functions ϕ_n and ϕ m, normalized in the region $0 \leq x \leq L$, where $n \neq m$ but are integers, are orthogonal. (5 marks)

iii). What is the physical significance of orthogonal wave functions? (2 marks)

b). i) Formulate the one dimensional problem of the transmission and reflection of an electron incident on a step potential barrier of height V. From it show that transmission is possible even when the electron energy $E < V$. (6 marks)

ii). Explain the concept of the perturbation theory. (2 marks)

iii). .If the one dimensional box energy equation were to be applied to an electron confined in a region of length 2Å and a 1g ball bearing in a 2cm long box, compare and explain the energy gap between $n = 1$ and $n = 2$ for the two. (4 marks)

Q5.

a

i). Write the quantum mechanical expression for the wave equation of a harmonic oscillator in one dimension with a restoring force Kx and state its energy operator. (3 marks)

ii). Hamiltonian operator is also known as the Energy operator, explain this (2 marks)

iii). Starting from the 3-Dimensional time-independent Schroedinger equation, obtain the Hamiltonian operator. (5 marks)

b) i). Define eigenfunction and give example (2 marks)

ii). Show that Ae-ikx I an eigenfunction under $\partial/\partial x$ but A Sin $a\pi x/L$ is not (3 marks)

iii). Define expectation value (2marks)

iv) Give expressions for the expectation values for position and momentum of a particle whose wave function is $\phi(x) = A \text{ Sin } a\pi x/L$ (3 marks)