KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SPH 400: CLASSICAL MECHANICS

DATE: FRIDAY, 25TH NOVEMBER 2011 **TIME: 8.00 A.M. – 10.00 A.M.**

INSTRUCTIONS: Answer question ONE and any other TWO (2) questions. Question ONE carries 30 marks while each of the others carries 20 marks.

1. (a) Show that the rotational kinetic energy T_{rot} of a rigid body is given as

$T_{rot} = \frac{1}{2}\omega.\mathbf{L}$, where $\boldsymbol{\omega}$ is the angular velocity of rotation of the rigid body about an axis passing through a fixed point and $\mathbf{L}$ is the angular momentum of the body about that fixed point. (6 marks)

(b) An inextensible string of negligible mass hanging over a smooth peg connects one mass m_1 on a frictionless incline plane of angle α to another mass m_2 which is hanging freely. Use the principle of virtual work to prove that the two masses will be in equilibrium if $m_2 = m_1 \sin\alpha$. (6 marks)

(c) Find the principal moments of inertia at the centre of a uniform rectangular plate of mass M and sides a and b. (Given that the moment of inertia about side a is $^1/_3 Mb^2$). (6 marks)

(d) In an Atwood's machine two masses M_1 and M_2 are connected by an inextensible string which passes over a fixed frictionless pulley of negligible mass. Apply

Lagrangian method to obtain the equation of motion for the Atwood's machine.

(6 marks)

(e) The Lagrangian for a simple pendulum is given as $L = \frac{1}{2}ml^2\dot{\theta}^2 + mgl\cos\theta$, where the terms have their usual meanings. Find the corresponding Hamiltonian and hence obtain the equation of motion of the simple pendulum. (6 marks)

2. (a) Show that the total angular momentum of a system of particles is conserved if the external torque acting on the system is zero and the internal forces obey strong law of action and reaction. (4 marks)

(b) If the internal and external forces acting on a system of particles can be derived from some scalar potentials, then show that the total energy of the system is conserved. (10 marks)

(c) Two particles of masses m_1 and m_2 are located on a frictionless double incline and connected by an inextensible massless string passing over a smooth peg. Use the D'Alembert's principle to obtain the equation of motion of the masses.

(6 marks)

3. (a) Considering the expression for angular momentum **L** of a body in terms of its moments of inertia and products of inertia, show that **L** is not necessarily always in the same direction as the instantaneous axis of rotation. (4 marks)

(b) Show that if a rigid body rotates about a principal axis, the direction of the angular momentum is the same as the principal axis of rotation. (6 marks)

(c) Given that the moment of inertia of a square plate of side a and mass M about a side is equal to $\frac{1}{3}Ma^2$ and the product of inertia about the two perpendicular sides is $-\frac{1}{4}Ma^2$, find the principal moments of inertia and the directions of the principal axes at the vertex of the square plate. (10 marks)

4. (a) Obtain, from the D'Alembert's principle, the Lagrange's equation of motion in terms of the kinetic energy of the system. (12 marks)

   (b) A bead is sliding on a uniformly rotating wire in a force-free space. The wire is straight and is rotating about a fixed axis perpendicular to the wire. Use the Lagrangian method to find the equation of motion of the bead.

   (8 marks)

5. (a) Define the Hamiltonian in terms of the Lagrangian and derive the Hamilton's canonical equations of motion. (10 marks)

   (b) Use the Hamilton's method to find the equation that describes the motion of a particle executing simple harmonic motion. (10 marks)

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