KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

SPH 400: CLASSICAL MECHANICS

DATE: MONDAY 2ND APRIL 2012 **TIME: 4.30 P.M. – 6.30 P.M.**

INSTRUCTIONS: Answer question **ONE** and any other **TWO** questions. Question ONE carries 30 marks and others each carries 20 marks

QUESTION ONE

a) (i) Distinguish linear and curvilinear motions. (2 marks)

(ii)State two conditions necessary for curvilinear motion. (2 marks)

(iii) Convert Cartesian coordinates P (2, 8) into Polar coordinates. (3 marks)

b) At an instant, the horizontal position of an object is described by $x = 8t$ meters, where t is time in seconds. If the equations of path is $y = \frac{x^2}{20}$; determine the:

(i) distance of the object from another point when t is 4 second (2 marks)

(ii) magnitude and direction of the velocity when t is 4 seconds (3 marks)

(iii) magnitude and direction of the acceleration when t is 4 seconds (4 marks)

c) (i) Distinguish linear momentum and angular momentum. (2 marks)

(ii) Explain term centre of mass and moment of inertia. (2 marks)

d) (i) State conservation law of angular momentum. (1 marks)

(ii) A particle moves in a force field of given by $\mathbf{F} = r^2\mathbf{r}$, where is the position vector of the particle. Prove that the angular momentum is conserved. (3 marks)

e) State the parallel axis theorem. (2 marks)

f) What is elastic collision? (1 mark)

g) What is Lagrangian equation? State its mathematical form explaining all the symbols used (3 marks)

QUESTION TWO

a) The position of a particle is described as (r, θ). If $\mathbf{r}_1$ is a unit vector in the direction of the position vector $\mathbf{r}$ and θ_1 is a unit vector perpendicular to $\mathbf{r}$ and in the direction increasing θ. Show that;

(i) $\mathbf{r}_1 = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta$ and $\theta_1 = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta$ (6 marks)

(ii) $\mathbf{i} = \mathbf{r}_1\cos\theta - \theta_1\sin\theta$ and $\mathbf{j} = \mathbf{r}_1\sin\theta + \theta_1\cos\theta$ (4 marks)

b) Prove that in polar coordinates (r, θ), the acceleration $\mathbf{a}$ is given by

$$a = (\ddot{r} - r\dot{\theta}^2)r_1 - (r\ddot{\theta} - 2\dot{r}\dot{\theta})\theta_1$$ (10 marks)

QUESTION THREE

a) (i) A mass of 5000 kg moves on a straight line from a speed of 540 km/hour to 720 km/hour in 2 minutes. Determine the impulse developed. (3 marks)

(ii) A certain system consists of n particles each with mass m. Using an arbitrary origin, derive an expression for centre of mass of the system. (5 marks)

b) Evaluate the centre of mass of a solid hemisphere of constant density ρ, mass m and radius r. (6 marks)

c) Obtain the Lagrange's equation of motion for a one-dimensional harmonic oscillator. (6 marks)

QUESTION FOUR

a) State D'Alembert's principle (2 marks)

b) Find the centroid of a uniform semicircular wire of radius r with σ as the density of mass per unit length (6 marks)

c) Show that the amount of inertial I_G of a uniform rod of length l and thickness t about the centre of gravity is given by

$$I_G = \frac{m(l^3 + r^3)}{12}$$

where the symbols have their usual meaning (12 marks)

QUESTION FIVE

a) Explain term "rigid body" and state the necessary and sufficient condition for rigid body to be in equilibrium (2 mark)

b) A simple pendulum having a bob of mass m is set in oscillation. If the length of string is l, find the Lagrangian·

(i) function L the simple pendulum. (6 marks)

(ii) equation describing the motion in Q5b(i) (4 marks)

c) Use the Hamiltonian method to find the equation of motion of a paqrticle of mass m constrained to move on the surface of a cylinder defined by $R^2 = x^2 + y^2$. The particle is directed to towards the origin and is proportional to the distance of the particle from the origin. (8 marks)