KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)

SPH 407: WAVEGUIDES

DATE: Tuesday, 29th November, 2011 TIME: 2.00 p.m. – 4.00 p.m.

INSTRUCTIONS: Answer question **ONE** and any other **TWO** questions.

You may use:

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m

$\mu_o = 4\pi \times 10^{-7}$ H/m

$$\nabla \times \nabla \times \vec{M} = \nabla(\nabla \cdot \vec{M}) - \nabla^2 \vec{M}$$

$$\nabla \times \vec{T} = \left(\frac{\partial T_z}{\partial y} - \frac{\partial T_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial T_x}{\partial z} - \frac{\partial T_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{a}_z$$

$$\nabla \times \vec{P} = \left[\frac{1}{r}\frac{\partial P_z}{\partial \theta} - \frac{\partial P_\theta}{\partial z}\right]\hat{r} + \left[\frac{\partial P_r}{\partial z} - \frac{\partial P_z}{\partial r}\right]\hat{\theta} + \left[\frac{A_\theta}{r} + \frac{\partial A_\theta}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right]\hat{z}$$

For rectangular waveguide

$$E_y = -\frac{ik}{k_c^2}\frac{\partial E_z}{\partial y} + \frac{i\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x} \qquad E_x = -\frac{ik}{k_c^2}\frac{\partial E_z}{\partial x} - \frac{i\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$H_y = -\frac{ik}{k_c^2}\frac{\partial H_z}{\partial y} - \frac{i\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial x} \qquad H_x = -\frac{ik}{k_c^2}\frac{\partial H_z}{\partial x} + \frac{i\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial y}$$

For circular waveguide

$$E_r = -\frac{1}{k_c^2}\left(\omega\mu\frac{H_z}{r} + ik\frac{\partial E_z}{\partial r}\right) \qquad E_\theta = \frac{1}{k_c^2}\left(i\omega\mu\frac{\partial H_z}{\partial r} - kn\frac{E_z}{r}\right)$$

$$H_\theta = -\frac{1}{k_c^2}\left(kn\frac{H_z}{r} + i\omega\varepsilon\frac{\partial E_z}{\partial r}\right) \qquad H_r = \frac{1}{k_c^2}\left(-ik\frac{\partial H_z}{\partial r} + \omega\varepsilon n\frac{E_z}{r}\right)$$

For parallel plate waveguide

$$H_y = -\frac{i\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \qquad E_x = -\frac{i}{h^2}\frac{\partial E_z}{\partial x} \qquad E_y = \frac{i\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \qquad H_x = -\frac{\gamma}{h^2}\frac{\partial EH_z}{\partial x}$$

$q_2^2 = n_2^2 k_o^2 - k^2$ $\quad$ $p_1^2 = k^2 - n_1^2 k_o^2$ $\quad$ $p_3^2 = k^2 - n_3^2 k_o^2$

Q1. (a) (i) Differentiate between a TM and a TE mode. (1 mark)

(ii) Sketch field pattern TE_{55} in a rectangular waveguide. (3 marks)

(b) (i) State one application of a dielectric waveguide. (1 mark)

(ii) Sketch field pattern TE_{20} in a parallel-plate waveguide. (3 marks)

(c) (i) What are degenerate modes in a rectangular waveguide. (1 mark)

(ii) Obtain the critical wave number for a 4 GHz wave propagating in a medium with $\mu_r = 1$ and $\varepsilon_r = 2.2$, if the phase shift (wave number) is 93.7 rad/m (3 marks)

(d) (i) Why is circular waveguide prefer in attenuation instruments than a rectangular waveguide. (1 mark)

(ii) Sketch the end-view field pattern for TE_{43} mode in a rectangular waveguide. (3 marks)

(e) (i) Give an example of a parallel plate transmission line. (1 mark)

(ii) Find the critical wave number of a lossless parallel-plate transmission line with a plate separation of 3 mm has a TM_1 waves propagating through it. (3 marks)

(f) (i) Draw a diagram illustrating excitation of TE_{10} in a rectangular waveguide using a coaxial cable. (1 mark)

(ii) Sketch the end-view field pattern for TE_{41} mode in a coaxial cable (3 marks)

(g)

Q2. (a) Using Maxwell's equations, show that the equation of a wave propagating in a rectangular wave guide is given by.

$$\nabla^2 E + \omega^2 \mu \varepsilon E = 0$$ (4 marks)

(b) Show that the equation

$$E_z = E_o \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\exp i[\omega t - kz]$$

satisfies the boundary conditions for TM_{mn} mode. (4 marks)

(c) Obtain the transverse fields (excluding E_z and H_z) for the TM_{11} modes of rectangular waveguide from the axial fields. (8 marks)

(d) A rectangular waveguide of dimensions a x b is filled with a dielectric material of permittivity $\varepsilon = k\varepsilon_o$. Show that for the TE_{10} mode the guide wavelength is given

$$\lambda_g = \frac{\lambda_o}{\sqrt{k - \left(\frac{\lambda_o}{\lambda_c}\right)^2}}$$

(4 marks)

Q3. (a) Using Maxwell's equations, show that the equation of a wave propagating a TEM mode in a coaxial line is given by.

$$\frac{E_\theta}{r} + \frac{\partial E_\theta}{\partial r} = 0$$

(4 marks)

(b) Show that the equation

$$E_\theta = \frac{E_o}{r}\exp i[\omega t - kz]$$

satisfies both sides of the propagation equation above. (4 marks)

(c) Using

$$E_z = E_0 J[k_c(a-b)\cos n\theta \exp i[\omega t - n\theta - kz]$$

obtain the transverse fields for the TM_{mn} modes in a coaxial line. (8 marks)

(d) An air-filled coaxial cable whose cut-off wavelength is 5 mm is propagating a TE_{mn}. If $k_c(\pi/2)(a+b) = 3.355$ and outer conductor radius is 1.6 mm, find the radius of the inner conductor. (4 marks)

Q4. (a) Using Maxwell's equations, show that the equation of a wave propagating in a parallel plate wave guide is given by.

$$\nabla^2 E + \omega^2 \mu\varepsilon E = 0$$

(4 marks)

(b) Show that the equation

$$E_y = (C_1 \sinh x + C_2 \cosh x)\exp[-\gamma z]$$

satisfies the equation

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0$$

for TE_{mn} mode. (4 marks)

(c) Obtain the transverse fields for the TE_{01} modes (8 marks)

(d) Calculate the minimum value of a, in rectangular waveguide of width a, operating in the TE_{10} mode over the frequency range over the frequency range from 10 to 11.5 GHz if the variation in v_g is not exceed 20 % of its value at 10 GHz. (4 marks)

Q5. (a) The solutions to TM waves propagating in the guiding layer (refractive index n_2) and cladding (refractive indices n_1 and n_2) of a dielectric wave guide are given by

$E_z(x) = A\sin(q_2 x) + B\cos(q_2 x)$ where $-h \le x \le 0$

$E_z(x) = C\exp(-p_1 x)$ where $x \ge 0$

$E_z(x) = D\exp(p_3[x+h])$ where $x \le -h$

The continuity boundary condition at x = 0 is B=C and at x = -h is $D = -A\sin(q_2 h) + B\cos(q_2 h)$. Show that the equation governing the modes is given by

$$\tan(q_2 h - m\pi) = \frac{q_2(r_1 p_1 + r_3 p_3)}{q_2^2 - r_1 r_3 p_1 p_3}$$

where m=1,2,3,...... and $r_i = (n_2/n_i)^2$ (12 marks)

(b) A TE_{10} signal at 10 GHz is propagating down a waveguide of width $a = 2$ cm. What is the change in phase velocity if the width is increased to 2.4 cm? (4 marks)

(c) Find the distance between two adjacent wave minima in a 2.3 cm x 1.0 cm rectangular air-filled waveguide which is propagating in the TE_{10} mode at the frequency at which the TE_{21} mode could just begin to propagate. (4 marks)

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