KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2007/2008

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SCE 118: CALCULUS FOR ENGINEERS

DATE: Monday 23rd June, 2008 **TIME**: 2.00 p.m. – 4.00 p.m.

INSTRUCTIONS:

Do any THREE (3) questions.

All questions carry equal marks.

Use clear illustrations where necessary.

Q1. (a) Determine and show clearly the domain and range of the following functions:

(i) $f(x) = \frac{1}{x^2}$ (ii) $f(x) = x^{3/2}$

(iii) $F(t) = \frac{1}{1+\sqrt{t}}$ (iv) $g(z) = \frac{1}{\sqrt{4 - z^2}}$

(b) Prove the following:

(i) $\tanh^{-1} = \frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right); |x| < 1$

(ii) $\mathrm{Coth}^{-1}x = \frac{1}{2}\log_e\left(\frac{x+1}{x-1}\right); |x| > 1$

(c) Sketch or plot the graph of $y = 3x^3 + 4x^2 - 13x + 3$ for $-4 \leq x \leq +2$ and hence solve the equations:-

(i) $3x^3 + 4x^2 - 13x + 6 = 0$

(ii) $3x^3 + 4x^2 - 13x + 17 = 0$

Q2. (a) Draw the graph of $y = x^2 - 4$ between $x = 4$ and $x = -4$. Draw on the same graph $y = 2x + 1$ and hence solve the simultaneous equations

$y = x^2 - 4$

$y = 2x + 1$

(b) Evaluate the limits that exist for

(i) $\lim\limits_{x \to 0} \dfrac{x^2 - 2x}{\sin 3x}$ (ii) $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{x(1 - \cos x)}$

(iii) $\lim\limits_{x \to 0} \dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$ (iv) $\lim\limits_{x \to 0} \dfrac{x^{1/3} - 1}{x^{1/2} - 1}$

(c) (i) Find the value of b for which the following function is continuous at x = 2:-

$$f(x) = \begin{cases} x, & -\infty < x < 0 \\ 1, & 0 \leq x < 2 \\ 3 - x, & 2 \leq x \end{cases}$$

Q3. (a) Differentiate the following from first principles:-

(i) $y = \sin x$ (ii) $y = \cos x$

(b) Given $\lim\limits_{\theta \to 0} \dfrac{\text{Sin}\,\theta}{\theta} = 1$, find

(i) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{5x}$ (ii) $\lim\limits_{x \to 0} \dfrac{\tan 2x}{5x}$

(iii) $\lim\limits_{t > \pi/2} \dfrac{\sin(t - \pi/2)}{t - \pi/2}$ (iv) $\lim\limits_{x \to 0} \dfrac{x^2}{\sec x - 1}$

(c) (i) Find dy/dx given $y = \dfrac{(1 + x^2)^{1/3}}{x(1 - x^2)^3}$

(ii) A certain damped oscillatory motion is given by

$S = Ke^{-pt} \sin (wt + \phi)$; where K, p, w are constants. Show that

$$\frac{d^2s}{dt^2} + 2p\frac{ds}{dt} + \left(w^2 + p^2\right)S = 0$$

(iii) The cost of producing x units of a machine parts in a factory is given by C = ($2x^3$ + 280x + 200) shillings. If each part is sold for (400 – 4x) shillings, calculate the number of units produced daily for the profit to be maximum and find the maximum daily profit.

Q4. (a) Evaluate the following integrals

(i) $\int \left(x^3 + 2\right)^2 3x^2 \, dx$ (ii) $\int \frac{\cos\theta}{\sin^4\theta} \, d\theta$

(iii) $\int \frac{3x + 2}{(x - 2)^2 (x + 1)} \, dx$ (iv) $\int \mathrm{Cos}^2 \, 2x \, dx$

(v) $\int \frac{1}{x^2 + 6x + 4} \, dx$

(b) Sketch the graph y = 10x – $5x^2$ above the x-axis. Then find the area between the curve and the x-axis, and the y-co-ordinate of the centroid of this area.

(c) Find the volume generated by rotating this area about the x-axis.

Q5. (a) Evaluate

(i) $\int_{-1}^{1} 3x^2 \sqrt{x^3 + 1} \, dx$

(ii) $\int_{\pi/4}^{\pi/2} \cot\theta \, \mathrm{Cosec}^2\theta \, d\theta$

(b) Use the trapezoidal rule with n = 4 to estimate

$$\int_1^2 x^2 \, dx$$

and compare it with the exact value.

(c) (i) Find the area of the region in he first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line y = x – 2.

(ii) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines y = 1, x = 4 about the line y = 1.

Q6. (a) Using the formula for a smooth curve

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$

from x = 0 to x = 2.

(b) Use the formula $S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$ to find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis.

(c) Find the volume of the solid of revolution described by the curve $y = \frac{1}{\sqrt{1 + x^2}}$ to generate the surface from $x = -\frac{\sqrt{3}}{3}$ to $x = \sqrt{3}$.

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