KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SMA 101: MATHEMATICS FOR SCIENCE II

DATE: Tuesday, 3rd April, 2012 TIME: 2.00 p.m. – 4.00 p.m.

INSTRUCTIONS: Answer question **ONE** and any other **TWO**.

QUESTION ONE (30 MARKS)

a) Evaluate the following limits

i. $\lim_{x \to 0}\left(\dfrac{\sin 2x}{x\cos 3x}\right)$ (3 marks)

ii. $\lim_{x \to \infty}\dfrac{8x+1}{\sqrt{x^2-12}}$ (3 marks)

iii. $\lim_{x \to -3}\dfrac{x^3+27}{x^2-9}$ (3 marks)

b) Differentiate the function $f(x)=\sqrt{2x+5}$ using first principles (5 marks)

c) Find the values of a and b that make f continuous

$$f(x)=\begin{cases} 2x & if\ x<1 \\ ax+b & if\ 1\le x\le 2 \\ 4x & if\ x>2 \end{cases}$$ (5 marks)

d) Differentiate the following functions with respect to x

i. $y=5x^2\cos^{-1}x$ (4 marks)

ii. $y=x^{\tan x}+3^x$ (4 marks)

e) A particle moves on the hyperbola $3x^2-y^2=12$ in such a way that its y coordinate increases at a constant rate of 9 units per second. How first is its x coordinate changing when $x=3$. (3 marks)

QUESTION TWO (20 MARKS)

a) i. State three conditions to be satisfied for a function to be continuous at find $x=a$ (3 marks)

ii. Determine whether the following function is continuous at $x = -1$ and at $x = 1$

$$f(x) = \begin{cases} -x+1 & if\ x < -1 \\ 2 & if\ -1 \le x \le 1 \\ -x+3 & if\ x > 1 \end{cases}$$ (8 marks)

b). Distinguish the stationary points of the curve $y = 8x^5 - 5x^4 - 20x^3$ .Hence sketch the graph (9 marks)

QUESTION THREE (20 MARKS)

a). Find the equation of the tangent and normal to the curve $x^2y^3 - y^2 + xy = 1$ at the point (1,1) (6 marks)

b). A bacterial colony is estimated to have a population of $P(t) = \dfrac{24t+10}{t^2+1}$ thousand t hours after the introduction of a toxin. Determine the time when the population is its greatest and compute the maximum population. (8 marks)

c). Given that $x = 1+t$, $y = 2t^2 - 4$ find $\dfrac{d^2y}{dx^2}$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate the following indefinite integrals

i. $\int \frac{1}{x^2}\left(x^4 - 2x^2 + 1\right)dx$ (3 marks)

ii. $\int \frac{x^2}{(2x^3+1)^{3/2}}dx$ (4 marks)

b) Differentiate the following functions with respect to x

i. $y = \sqrt{2 + \sqrt{4 + \sqrt{3x}}}$ (5 marks)

ii. $y = \left(\sqrt{x^2+1}\right)\left(x^2 + 2x + 1\right)^2$ (4 marks)

c) Find the area under a curve $y = 1 - \sqrt[3]{x}$ from $x = -8$ to $x = -1$ (4 marks

QUESTION FIVE (20 MARKS)

a) Use $y = x^{2/3}$ to approximate $(122)^{2/3}$. (6 marks)

b) Evaluate the following limits

i. $\lim\limits_{x \to 5} \left(\dfrac{25 - x^2}{x^2 - 10x + 25} \right)$ (3 marks)

ii. $\lim\limits_{x \to \infty} \left(\dfrac{6x^3 + 2x^2 - 4x + 2}{3x^3 + 3x - 6} \right)$ (2 marks)

iii. $\lim\limits_{x \to 2} \left(\dfrac{\sqrt{x^2 + 5} - 3}{x^2 - 2x} \right)$ (4 marks)

c) The concentration of a certain drug in a patient blood stream t hours after injection is given by $C(t) = \dfrac{0.2t}{t^2 + 1}$ mg/cm^3 .Evaluate $\lim\limits_{x \to \infty} C(t)$ and interpret your result.

(5 marks)