



KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2009/2010
INSTITUTIONAL BASED PROGRAMME (IBP)
EXAMINATION FOR THE DEGREE BACHELOR OF SCIENCE

SMA 103: ANALYTIC GEOMETRY

DATE: Friday 30th April 2010

TIME: 11.00a.m – 1.00p.m

INSTRUCTIONS: Answer Question ONE and any other TWO questions

1. a) Find the rectangular equation of the curve whose polar equation is

$$r = \frac{8}{1 + 4\cos\theta} \quad [4\text{marks}]$$

- b) Find the slope of a line which is perpendicular to the line joining

$$P_1(2,4), P_2(-2,1). \quad [2\text{marks}]$$

- c) Find the acute angle between the following lines L_1 and L_2 whose equation are $8x + 3y - 5 = 0$ and $3x - 2y + 8 = 0$. [5marks]

- d) Show that the two circles $x^2 + y^2 - 3x + 2y - 3 = 0$ and $x^2 + y^2 + 2x + y + 1 = 0$ are orthogonal [7marks]

- e) Express the following equation $2x^2 + 2y^2 - 5x + 4y - 7 = 0$ in standard form, hence write down the coordinates of the centre and the length of the radius. [3marks]

- f) Find the equation of the ellipse with vertices at $(5, 1)$ and $(-1, 1)$ and foci at $(4, 1)$ and $(0, 1)$. [5marks]

- g) Write the equation of the tangent and the normal to the hyperbola $16x^2 - 9y^2 - 128x - 54y + 31 = 0$ at $(\frac{1}{4}, 0)$ [4marks]

2. a) Find the equation of the tangent to the circle $x^2 + y^2 + 4x - 5y + 9 = 0$ at $(-1, 3)$. [5marks]
- b) Find the length of a tangent to the circle $x^2 + y^2 - 6x + 2y - 6 = 0$ from A $(-2, 0)$. [4marks]
- c) Find the equation of a circle which passes through $(2, -1)$ and $(-2, 0)$ with the centre on the line $2x - y - 1 = 0$. [11marks]
3. a) Given that the vertex of a parabola is $V(3, 5)$ and its directrix is the line $y = -4$, determine the equation of the parabola and find its focus. Hence sketch the curve. [4marks]
- b) Write the equation of the tangent and normal to the parabola $y^2 - 12x - 2y - 23 = 0$ at $(1, 7)$ [8marks]
- c) Express the following equation in standard form, hence write the coordinates of vertex and focus, the equations of the directrix and axis.
 $4x^2 + 4x + 4y + 9 = 0$. [8marks]
4. a) What is an ellipse? [2marks]
- b) An ellipse has foci $F_1(0, -c)$ and $F_2(0, c)$ with constant sum $2a$. If $P(x, y)$ is any point on the ellipse, show that $\frac{y^2}{a^2} + \frac{x^2}{a^2 - c^2} = 1$ [6marks]
- c) Express the following equation of an ellipse in standard form.
 $7x^2 + 4y^2 - 14x + 40y + 79 = 0$.
- i) Compute the semi major and semi minor axes and the eccentricity. [5marks]
- ii) Write the coordinates of the centre, vertices and foci. [5marks]
- iii) Write the equations of the directrices [2marks]

5. a) The following is equation of a hyperbola $16x^2 - 9y^2 - 128x - 54y + 31 = 0$
- i) Find the coordinates of the centre, vertices and foci. [11marks]
 - ii) Find the lengths of the major tranverse axis [1mark]
 - iii) Find the equations of the directrices. [3marks]
 - iii) Sketch the curve. [1mark]
- b) Write down the equation of the hyperbola with one vertex at (5, -4) and with asymptotes $y + 4 = \pm \frac{3}{2}(x - 3)$ in standard form. [4marks]