



KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2010/2011
OPEN, DISTANCE AND E-LEARNING EXAMINATION FOR THE DEGREE
OF BACHELOR OF SCIENCE

SMA 104: CALCULUS I

DATE: Wednesday 6th July 2011 **TIME:** 2.00p.m – 4.00p.m

INSTRUCTIONS: Answer Question ONE and any other TWO Questions.

QUESTION ONE (30MARKS)

- a) Evaluate the following limits.

i) $\lim_{x \rightarrow 1} \frac{x^2 + 6x - 7}{x - 1}$ [2]

ii) $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ [3]

- b) Use the First principle to find the derivative of the function

$y = \frac{1}{x^3}$ with respect to x . [5]

- c) Use chain rule to differentiate

$y = h(x) = (x^2 - 6x + 5)^3$ [3]

- d) Use product rule to differentiate

$y = h(x) = e^{2x^3}(x^2 + x + 2)$ [3]

- e) Given the function $f(x) = x^4 - 2x^2 + 7$. Find the turning points. [5]

- f) The position of a particle on a line is given by $s(t) = t^3 - 3t^2 - 6t + 5$, where t is measured in seconds and s is measured in feet. Find

i) The velocity of the particle at the end of 2 seconds. [2]

ii) The acceleration of the particle at the end of 2 seconds. [2]

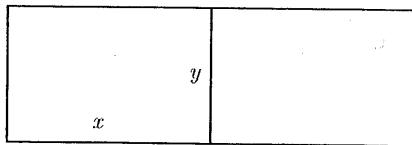
- g) Find the equation of the normal line to the parabola $y = 4x^2$ at the point $(-1, 4)$ [5]

QUESTION TWO (20MARKS)

- a) Find the derivative of $y = e^{\sqrt{x^3+4}}$ [3]
- b) Find the equation of the tangent and normal at a point for which $t = 2$, given that the parametric equations at a curve are $x = \frac{3t}{1+t}$, $y = \frac{t^2}{1+t}$ [7]
- c) Find the turning points of the curve $y = x^4 - 6x^2 + 8x + 10$ distinguish them, hence sketch the curve. [10]

QUESTION THREE (20MARKS)

- a) Use implicit differentiation to evaluate $x^2 + y^2 - 2x - 6y + 5 = 0$ [3]
- b) Find $\frac{dy}{dx}$ given that $y = \sqrt{\frac{2x+1}{3x-2}}$ [6]
- c) If $y = \sqrt{1+\sin x}$, show that $\frac{dy}{dx} = \frac{1}{2}\sqrt{1-\sin x}$ [5]
- d) We have 1200 metres of fencing material, and wish to enclose a double paddock with two equal rectangular areas as shown in the diagram below.



Suppose that each of the two rectangular areas has sides x and y in metres, as shown in the picture. Find the dimensions of x and y that will maximize the area enclosed by the fence. [6]

QUESTION FOUR (20MARKS)

- a) Find $\frac{dy}{dx}$ for the function $y = (x-1)\sqrt{x^2 - 2x + 2}$ [3]
- b) Evaluate the limit
$$\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$$
 [5]
- c) Gas is escaping from a spherical balloon at the rate of $900\text{cm}^3/\text{sec}$. How fast is the surface area shrinking when the radius is 360cm^2 ? [5]
- d) A closed rectangular container has a square base and is required to have a volume of 64cm^3 , if the container is made of thin metal find the dimensions which will minimize the surface area. [7]

QUESTION FIVE (20MARKS)

- a) Find the derivative of the function $y = x^3 \sin(3x+2)$. [3]
- b) If $y = e^{3x} \sin 4x$. Find
$$\frac{d^2 y}{dx^2} - \frac{dy}{dx} + 25y$$
 [7]
- c) A rental agent estimates that the monthly profit p from a building 5 storeys high is given by $p = 4600s - 100s^2$. What height would maximize the profitable? [4]
- d) Determine the area of the largest piece of rectangular ground that can be enclosed by 100m of fencing, if part of an existing wall is used as one side. [6]