



KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 104: CALCULUS 1

DATE: Thursday 5th April, 2012

TIME: 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS: Attempt Question One and any other TWO.

Question 1 (30 Marks)

a.

- i) Consider the function $f: x \rightarrow x^2 + 1, x \in \mathbb{R}$. Does it have an inverse?

Explain.

[3 marks]

- ii) Hence or otherwise find the inverse of the function $f: x \rightarrow x^2 + 1, x \leq 0$.

[3 marks]

- b. Find the equation of the tangent line $f(x) = 15 - 2x^2$ at $x = 1$

[5 marks]

- c. A particle of mass m moves along the x -axis. The velocity v and position x satisfy the equation $m(v^2 - v_0^2) = k(x_0^2 - x^2)$ where k, v_0 and x_0 are constants.

Show by differentiating implicitly with respect to t that whenever $v \neq 0$,

$$m \frac{dv}{dt} = -kx$$

[5 marks]

- d. Given that $f(x) = \begin{cases} 9, & \text{if } x < 3 \\ 1, & \text{if } x = 3 \\ (x - 9)^2, & \text{if } x > 3 \end{cases}$

- i) Evaluate $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$.

[2 marks]

- ii) Is the function continuous at $x = 3$? Explain.

[2 marks]

- e. Evaluate the following limit:

$$\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x).$$

[5 marks]

- f. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 6xy$. Simplify your answer completely.

[5 marks]

Question 2 (20 Marks)

- a. Define the functions f, g and h by $f(x) = x^4$, $g(x) = \sqrt{x^2 + 1}$ and $h(x) = x^2 + 72$.
Find $h \circ g \circ f$. [2 marks]

- b. Functions f and g are defined by:

$$f : x \rightarrow 2x - 5, x \in \mathbb{R}$$

$$g : x \rightarrow \frac{4}{2-x}, x \in \mathbb{R}, x \neq 2$$

- i) Find the value of x for which $f \circ g(x) = 7$ [2 marks]
- ii) Find $(g \circ f)^{-1}(3)$ [3 marks]
- c. The parametric equations of a curve are $x = 8 \cos t$, $y = 8 \sin t$. Find the equation of the tangent to the curve at the point where $t = \frac{\pi}{3}$. [5 marks]

- d. Differentiate the following functions with respect to x .

i) $y = x^2 \cos^{-1}(3x^2 + 2)$ [2 marks]

ii) $y = x^{\sqrt{x}}$ [4 marks]

iii) $y = \sinh^{-1}(\ln x)$ [2 marks]

Question 3 (20 Marks)

- a. Evaluate :

$$\lim_{x \rightarrow -3^+} \frac{x+2}{x+3}$$

[2 marks]

- b.

- i) Find the stationary points of $3x^4 - 8x^3 + 6x^2 - 3$ and investigate their nature. [7 marks]

- ii) Sketch the curve. [3 marks]

- c. A spherical balloon is being blown up so that its volume increases at a constant rate of $1.5 \text{ cm}^3 / \text{s}$. Find the rate of increase of the radius when the volume of the balloon is 56 cm^3 . [5 marks]

- d. Using $y = \sqrt[3]{x}$, find the approximate value for $y = \sqrt[3]{124}$. [3 marks]

Question 4 (20 Marks)

- a. Find $\frac{dy}{dx}$ using the logarithmic function given that $y = \sqrt{x(x+1)}$. Simplify your answer completely. [3 marks]
- b. A box with a square base is open at the top. If 108cm^3 of material are used, what is the maximum volume possible for the box? [4 marks]
- c. Find the equation of the normal line to $x^2 + y^2 = 9$ at the point $(2, \sqrt{5})$. [6 marks]
- d. Differentiate the function $h(t) = \frac{\sinh t}{t+1}$ with respect to t . [2 marks]
- e. Find $\frac{dy}{dx}$ for each of the following:
- i) $x^2 \tan y + y^{10} \sec x = 2x$ [2 marks]
- ii) $e^{2x+3y} = x^2 - \ln(xy^3)$ [3 marks]

Question 5 (20 Marks)

- a. If $e^x y = \sin x$ show that $\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$. [5 marks]
- b. A body rises vertically from the earth according to the law $s = 19.6t - 4.9t^2$. Show that it has lost half of its velocity in its first 14.7m of rise. [5 marks]
- c. Using the first principles find the derivative of $f(x) = \sin 3x$. [5 marks]
- d. Let $f(x) = \begin{cases} x^4, & x \leq 1 \\ ax + b, & x > 1 \end{cases}$ where a and b are constants. Calculate the values of a and b in order that $f(x)$ together with $f'(x)$ are continuous at $x = 1$. [5 marks]

