KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010
INSTITUTIONAL BASED PROGRAMME
EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

SMA 200: CALCULUS II

DATE: **Wednesday 1st September 2010** **TIME:** **8.00a.m – 10.00a.m**

INSTRUCTIONS:

Answer question **ONE** and any other **TWO**.

Q1. - [Compulsory - 30 marks]

(a) Find the following indefinite integrals: (3 marks each)

i. $\displaystyle\int \frac{x^2+1}{x^2}dx$

ii. $\displaystyle\int \frac{x^2}{\sqrt{x^3+3}}dx$

(b) Evaluate the following definite integrals: (3 marks)

i. $\displaystyle\int_0^{\frac{\sqrt{3}}{2}} \frac{dx}{\sqrt{1+4x^2}}$

ii. $\displaystyle\int_1^2 x^2(\ln x)dx.$

iii. $\displaystyle\int_0^4 \frac{x}{\sqrt{2x+1}}dx.$

(c) Find the area bounded by the graph $y = 2x^2 - 3x + 2$, the x-axis and the ordinates x=0, x= 2. (3 marks)

(d) Find the area of the surface swept out when the curve $y = \sqrt{x}$ between x = 1 and x = 4 rotates about the x-axis. (3 marks)

(e) Find the volume of the solid generated by revolving the region bounded by the graph $f(x) = \sqrt{\sin x}$ for $0 \le x \le \pi$ and y=0 about the x-axis. (4 marks)

(f) Show that the function $f(x) = \int_0^{\frac{1}{x}} \frac{du}{u^2+1} + \int_0^{x} \frac{du}{u^2+1}$ is a constant for $x > 0$. (3 marks)

(g) Evaluate the improper integral $\int_1^4 \frac{1}{x^2} dx$. (2 marks)

Q2. [Optional-20 marks]

(a) Use the limiting formula $\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(a+i\frac{b-a}{n})$ for the area under the curve to evaluate the area bounded by the curve $f(x) = x^2+1$, the x-axis and the vertical lines $x = 0$ and $\cancel{x23}$. Confirm your answer by direct integration. (10 marks)
$x=3$

(b) Find the area of the region bounded by the graphs of ~~the~~ $f(x) = x^2+2x+1$ and $g(x) = 3x+3$. (5 marks)

(c) Find the volume of the solid generated by resolving the region bounded by curves, $y = x\sqrt{4-x^2}$ and $y = 0$ about the y-axis. (5 marks)

Q3. [Optional - 20 marks]

(a) Find the length of the arc of the curve $f(x) = x^{\frac{3}{2}} - 1;\ 0 \le x \le 4$. (6 marks)

(b) Find the surface area of the solid generated by revolving the curve $y = \frac{x^3}{6} + \frac{1}{2x}; 1 \le x \le 2$, about the x-axis. (7 marks)

(c) A sphere of radius r is generated by revolving the graph the graph of $y = \sqrt{r^2-x^2}$ about the x-axis. Verify that the surface area of the sphere is $4\pi r^2$. (7 marks)

Q4. [Optional- 20 marks]

(a) Evaluate the following integrals:

i. $\displaystyle\int \frac{1}{x\sqrt{4x^2-1}}dx.$ (4 marks)

ii. $\displaystyle\int e^{2x}\sin 2x dx.$ (4 marks)

iii. $\displaystyle\int \frac{\cos^3 x}{\sqrt{\sin x}}dx.$ (4 marks)

(b) Use partial fractions to evaluate the integral

$$\int \frac{x^2+12x+12}{(x^3-4x)}dx. \quad \text{(8 marks)}$$

Q5. [Optional-20 marks]

(a) Using the First Fundamental Theorem of Calculus evaluate $F'(x)$ if $F(x) = \displaystyle\int_0^{x^3} \sin t^2 dt$ (3 marks)

(b) show that with the substitution $u = x - \frac{1}{x}$ the integral

$$I = \int_1^{\sqrt{2}} \frac{x^2+1}{x^4+1}dx$$

becomes

$$I' = \int_0^{\frac{1}{\sqrt{2}}} \frac{1}{u^2+2}du. \quad \text{(4 marks)}$$

(c) Use the Simpson's rule with n = 6 to approximate the integral

$$\int_0^2 \frac{1}{\sqrt{1+x^3}}dx. \quad \text{(7 marks)}$$

(d) Determine the smallest value of n so that Trapezoidal rule will approximate the value of the integral

$$\int_0^1 \frac{1}{1+x}dx.$$

with an error less than 0.00001. (6 marks)