KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF

BACHELOR OF SCIENCE

SMA 201: CALCULUS III

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DATE: WEDNESDAY 28TH MARCH 2012 TIME: 8.00 A.M. – 10.00 A.M.

INSTRUCTIONS

Answer question ONE and any other TWO questions

Question One (30 marks)

1. a) Evaluate

$$\lim_{x\to 0}\left(\frac{3x+1}{x}-\frac{1}{\sin x}\right)$$ (3 marks)

b) Find the Maclaurin's series for $f(x)=\dfrac{\cos x}{(1+x)^2}$ up to the term containing x^4 (3 marks)

c) Verify Rolle's Theorem for the function's $f(x)=x^3-3x^2+x+1$

On the interval $\left[1\ ,\ 1+\sqrt{2}\right]$. (4 marks)

d) i) Find $\dfrac{\partial z}{\partial r}$ given that $z=f(x\ ,\ y)$, $x=g(r,\ s)$, $y=h(r,s)$.

(1 mark)

ii) Given the simultaneous function

$F(x, y, u, v) = 0$, $G(x, y, u, v) = 0$, find the expression for $\dfrac{\partial u}{\partial x}$ (3 marks)

e) If $z = f(x, y) = x^2 y - 3y$

i) Find dz given that $x = 4$, $y = 3$, $\Delta x = -0.01$, $\Delta y = 0.02$ (2 marks)

ii) Determine $f(5.12, 6.85)$ without direct computation. (4 marks)

f) State Green's Theorem in the plane and use it to evaluate

$\oint (xy + y^2)\, dx + x^2\, dy$

Where C is a closed curve of the region bounded by $y = x$ and $y = x^2$

(6 marks)

g) Use stokes theorem to evaluate

$\int_c F \bullet dr$ if $F = y\underset{\sim}{i} - x\underset{\sim}{j}$ and S

is the hemisphere $S : x^2 + y^2 + z^2 = 9$, $z \geq 0$, its boundary circle $x^2 + y^2 = 9$, $z = 0$. (4 marks)

<u>Question Two (20 marks)</u>

2. a) Sketch the region bounded by the curve $y = x^2$, $x = 2$, $y = 1$ and hence evaluate $\iint (x^2 + y^2)\ dx\, dy$ over the region (8 marks)

b) A rectangular tank, open at the top is to have a volume of 32 m$^{3.}$ Determine the dimensions so that the total surface area is a minimum. Calculate the cost of chrome dating the inside of the tank if it costs Ksh.640.00 per m^2 (12 marks)

<u>Question Three (20 marks)</u>

a) Evaluate the following limits

i) $\lim\limits_{x \to \frac{\pi}{4}} (1 - \tan x) \sec 2x$ (2 marks)

ii) $\lim\limits_{x \to 0} \left(e^{x} + 3x\right)^{1/x}$ (3 marks)

iii) $\lim\limits_{x \to \infty} \dfrac{(\ln x)^2}{x^3}$ (3 marks)

b) Find the Maclaurin's series for the function $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$ upto the term containing x^5 (5 marks)

c) Find the values of x for which *sin x* can be replaced by $x - \dfrac{x^3}{3!}$ with an error magnitude no greater than 3.0×10^{-4} (4 marks)

d) Compute the first four non-zero terms of the Taylor's series for $f(x) = e^{\frac{1}{2}x+1}$ in ascending power of *(x - 2).* (3 marks)

Question Four (20 marks)

a) Given that $z = x^3 \tan^{-1}\left(y/x \right)$, find $\dfrac{\partial^2 z}{\partial x \partial y}$ at $(1, 1)$. (5 marks)

b) Show that $y = f(x + at) + g(x - at)$ satisfies Laplace's equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$ (6 marks)

c) Apply the Mean Value Theorem to the function $f(x) = x^3 - x^2 - 2x$ on the interval $[-1, 1]$. (4 marks)

d) Evaluate the triple integral $\int_0^2 \int_0^x \int_0^{x+y} e^x (y + 2z)\, dz\, dy\, dx$. (5 marks)

Question 5

a) i) State Stokes Theorem (2 marks)

ii) Verify Stokes theorem for $\underset{\sim}{A} = (2x - y)\underset{\sim}{i} - yz^2 \underset{\sim}{j} - y^2 z \underset{\sim}{k}$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$

and C is its boundary. (8 marks)

b) i) Show that the area bounded by a simple closed curve C is given by

$\frac{1}{2} \oint xdy - ydx$ (4 marks)

ii) Use Green's theorem to evaluate

$\oint_c (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where C is a square with vertices at

$(0, 0)$, $(2, 0)$, $(2, 2)$ $(0, 2)$ (6 marks)