KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 202: LINEAR ALGEBRA I

DATE: FRIDAY, 30TH APRIL 2010 **TIME: 8.00 A.M. - 10.00 A.M.**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (30 marks)

(a) If $\begin{vmatrix} x & 3 \\ 2 & 4x-1 \end{vmatrix} = -1$, find x. (4 marks)

(b) Find the dot and the cross products of the vectors

$\mathbf{u} = (3, 2, 7)$ and $\mathbf{v} = (2, 4, -2)$ (4 marks)

(c) Evaluate $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{vmatrix}$ (2 marks)

(d) Use Cramer's rule to solve

$$\begin{aligned} 2x - 5y + 2z &= 7 \\ x + 2y - 4z &= 3 \\ 3x - 4y - 6z &= 5 \end{aligned}$$

(4 marks)

(e) Determine whether or not the following is a linear transformation

$T(x,y,z) = (3x - y,\ 2x + z,\ x + 2z)$ (4 marks)

Subspace

(f) Let $V = \Re^3$. Show that $U = \{(x, y, 0): x, y \in \Re\}$ is a subset of V. (3 marks)

(g) Write the vector $\mathbf{v} = (1, -2, 5)$ as a linear combination of the vectors

$\mathbf{e}_1 = (1, 1, 1)$, $\mathbf{e}_2 = (1, 2, 3)$ and $\mathbf{e}_3 = (2, -1, 1)$. (5 marks)

(h) Determine whether or not the following vectors in $\Re^3$ are linearly dependent. (1, –2, 1), (2, 1, –1), (7, –4, 1) (4 marks)

QUESTION TWO (20 marks)

(a) Find m such that the vector $\mathbf{u} = (1, -2, m)$ in $\Re^3$ is a linear combination of $\mathbf{v} = (3, 0, -2)$ and $\mathbf{w} = (2, -1, -5)$. (5 marks)

(b) Find the angle between the vector $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = -\mathrm{i} + 3\mathbf{j} - \mathbf{k}$. (4 marks)

(c) For any three vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$, show that $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$. (5 marks)

(d) Determine whether or not the matrices

$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ *and* $C = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ are linearly independent. (6 marks)

QUESTION THREE (20 marks)

(a) Solve the following equations using matrix inversion method.

$$\begin{aligned} x + y + z &= 5 \\ x + 2y + 3z &= 10 \\ 2x + y + z &= 6 \end{aligned}$$

(9 marks)

(b) Find x, y and z if $(2,-3,4) = x(1,1,1) + y(1,1,0) + z(1,0,0)$. (5 marks)

(c) Let **a**, **b** and **c** be vectors. Show that $\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Where $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j} + c_3\mathbf{k}$.

(6 marks)

<u>QUESTION FOUR</u> (20 marks)

(a) Find a basis and the dimension of the subspace **W** of $\Re^4$ spanned by (1, –4, –2, 1), (1, –3, –1, 2) and (3, –8, –2, 7). Extend the basis of **W** to a basis of the whole space $\Re^4$. (5 marks)

(b) Show whether or not the following are subspaces of $\Re^3$.

(i) **U** = {(x, y, z) | x = 2y} (3 marks)

(ii) **V** = {(x, y, z) | $x^2 + y^2 + z^2 \leq 1$} (3 marks)

(c) Find the conditions on a, b and c so that (a, b, c) $\in \Re^3$ belong to the space generated by **u** = (2, 1, 0), **v** = (1, –1, 2) and **w** = (0, 3, –4).

(5 marks)

(d) Let **U** be the subspace **W** = {(a, b, c, d) | a + b = 0, c = 2d} of $\Re^4$. Find the dimension and a basis of **U**. (4 marks)

<u>QUESTION FIVE</u> (20 marks)

(a) Find the angle between the planes $2x + y - 2z = 1$ *and* $x - 2y - 2z = 2$.

(4 marks)

(b) Find the equation of the plane through the points A(1, 1, 1), B(2, 0, 2) and C(–1, 1, 2). (6 marks)

(c) Define a function T: $\Re^2 \rightarrow \Re^3$ by T(x, y) = (x + y, x – 2y, 3x) for all (x, y) $\in \Re^2$. Show that T is a linear transformation. (4 marks)

(d) Let T: $\mathfrak{R}^4 \to \mathfrak{R}^3$ be a linear transformation defined by

T(x, y, z, w) = (x – y + z + w, x + 2z – w, x + y + 3z – 3w). Find a basis and the dimension of the

(i) image of T (ii) kernel of T (6 marks)

END OF EXAMINATION