KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2011/2012

MOMBASA CAMPUS

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE

SMA 203: LINEAR ALGEBRA II

DATE: Thursday 1st December 2011 **TIME: 2.00p.m – 4.00p.m**

INTRUCTIONS:

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY-30MKS)

(a) (i) State the Cayley-Hamilton Theorem. (2mks)

(ii) Show that $A=\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$ is a zero of $f(t)=t^2-4t-5$. (2mks)

(b) Find the determinant of $A=\begin{pmatrix}1 & 0 & 0 & 3\\ 0 & 1 & -2 & 0\\ -2 & 3 & -2 & 3\\ 0 & -3 & 3 & 3\end{pmatrix}$. (3mks)

(c) Let P_n denote the vector space of all polynomials in x of degree less than or equal to n over $\Re$. Let $T: P_2 \rightarrow P_3$ be the linear mapping defined by $T[P(x)] = xP(x-3)$ i.e. $T(a_0+a_1x+a_2x^2) = x(a_0+a_1(x-3)+a_2(x-3)^2)$ .Find the matrix representation of T with respect to the bases $e=\{1,x,x^2\}$ and $f=\{1,x,x^2,x^3\}$. (4mks)

(d) (i) Prove that similar matrices have the same determinant. (4mks)

(ii) Let T: $\Re^2 \rightarrow \Re^2$ be defined by $T(x,y)=(2x-y,-3x+2y)$ .Find the determinant of T. (3mks)

(e) Let $M_A(x)=x^2-5x+6$ be a minimal polynomial of matrix A. Show that $A^5=211A-390I$. (3mks)

(f) (i) What do you understand by the term "Trace of a $n \times n$ matrix"? (1mk)

(ii) Let be defined by $T(x,y,z)=(2y+z,x-4y,3x+7y)$. Find trace T under the basis $\{f_i\}=\{(1,1,1),(1,1,0),(1,0,0)\}$. (4mks)

(g) Let V be a vector space of 2×2 matrices over $\Re$. Find the coordinate vector of the matrix $A\in V$ relative to the basis $\left\{\begin{pmatrix}1&1\\1&1\end{pmatrix},\begin{pmatrix}0&-1\\1&0\end{pmatrix},\begin{pmatrix}1&-1\\0&0\end{pmatrix},\begin{pmatrix}1&0\\0&0\end{pmatrix}\right\}$ where $A=\begin{pmatrix}4&6\\8&-14\end{pmatrix}$. (4mks)

QUESTION TWO (20MKS)

(a) Let T be the linear mapping T: $\Re^2\to\Re^2$ defined by $T(3,1)=(2,-4)$ and $T(1,1)=(0,2)$. Find (x,y) and hence $T(6,5)$. (5mks)

(b) Consider the following bases of $\Re^3$:

The standard basis and $\{f_1=(1,-1,0), f_2=(1,0,-1), f_3=(0,0,1)\}$

Verify that

(i) $P^{-1}[v]_e=[v]_f$ for any vector $v\in\Re^3$. (4mks)

(ii) The matrix representation $[T]_f=P^{-1}[T]_e P$ where is T defined by $T(x,y,z)=(-2x+y+z,x-2y+z,x+y-2z)$. (7mks)

(c) Find the value of t for which the determinant of A is zero if

$$A=\begin{pmatrix}t-2&4&3\\1&t+1&-2\\0&0&t-4\end{pmatrix}.$$ (4mks)

QUESTION THREE (20MKS)

(a) Let T: $\Re^3\to\Re^2$ be the linear mapping which defined by $T(x,y)=(3x+2y-4z,x-5y+3z)$.

(i) Find the matrix representation of T relative to the following bases of $\Re^3$ and $\Re^2$:

$\Re^3$: $\{e_1=(1,1,1), e_2=(1,1,0), e_3=(1,0,0)\}$ and $\Re^2$: $\{g_1=(1,3), g_2=(2,5)\}$. (4mks)

(ii) Verify that $[T]_f^g[v]_f=[T(v)]_g$. (5mks)

(b) Let $T(x,y)=(2x+4y,x-2y)$ and let the standard basis and $f_1=(1,1),f_2=(2,-1)$ be two bases of $\Re^2$. Is T diagonalizable? (7mks)

(c) Let $A=\begin{pmatrix}1 & 3 & -1\\ 2 & 0 & 5\\ 6 & -2 & 4\end{pmatrix}$ be a matrix representation of $T:P_2\to P_2$ with respect to the basis $B=\{v_1,v_2,v_3\}$ where $v_1=3x+3x^2$, $v_2=-1+3x+2x^2$, $v_3=3+7x-2x^2$. Find $T(v_1)$, $T(v_2)$ and $T(v_3)$. (4mks)

QUESTION FOUR (20MKS)

(a) Find the eigenvalues and a basis for the eigenspace for each eigenvalue if

$$A=\begin{pmatrix}2 & 1 & 0\\ 0 & 1 & -1\\ 0 & 2 & 4\end{pmatrix}.$$ (6mks)

(b) Let A and B be two similar matrices. Show that A and B have the same characteristic polynomial. (4mks)

(c) Find the characteristic and minimal polynomials of A given that

$$A=\begin{pmatrix}3 & 1 & 0 & 0\\ 0 & 3 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}.$$ (5mks)

(d) Let V be the space of 2X2 matrices over $\Re$ and let $M=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$. If T is a linear operator on V defined by $T(A)=MA$, find trace of T. (5mks)

……………………………………………………………END……………………………………………………………