KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 204: ALGEBRAIC STRUCTURES

DATE: THURSDAY, 25TH NOVEMBER 2010 **TIME: 8.00 A.M. - 10.00 A.M.**

INSTRUCTIONS: Answer question ONE and any other TWO questions.

QUESTION 1 (30 marks)

a) Define $f:\Re\to\Re$ by $f(x)=\dfrac{1}{x-4}$. Determine the domain, range and $f^{-1}(x)$. (6 marks)

b) Let $f:\mathbf{Z}\to\mathbf{Z}$, g: $\mathbf{Z}\to\mathbf{Z}$ defined by $f(n)=n^2+2n+1$, $g(n)=4n-5$. Show that $(g\circ f)(n)\neq(f\circ g)(n)$ (4 marks)

c) Let $*$ be a binary operation defined on $\Re$ by $x*y=xy-x-y$. Show that $*$ is commutative but not associative. (4 marks)

d) Let $*$ be a binary operation defined on a non-empty set A, with identity. prove that the identity element is unique. (4 marks)

e) Suppose $\langle G,*\rangle$ is a group whose elements are $\{e,a,a^2,a^3,a^4,a^5\}$ with $a^6=e$. Draw the Cayley table of G. Determine the inverse of each element and all subgroups of G. (7 marks)

f) Differentiate between a **zero divisor** and a **unit** in a ring $\langle R,+,\cdot\rangle$. Determine all zero divisors and units in $\langle Z_{18},+,\cdot\rangle$. (5 marks)

QUESTION 2 (20 marks)

a) Define $f:\Re \to \Re$ by $f(x) = \dfrac{x}{x+1}$

(i) Find $f(-5)$, the domain and range of f. (4 marks)

(ii) Find $f^{-1}(x)$ and $f^{-1}(3)$ (3 marks)

(iii) Show that f is injective (3 marks)

b) Let $A = \{a,b,c,d,e,f\}$, $B = \{3,5,7,10,13,15,18\}$ and $D = \{p,q,r,s,t,u,v,w\}$.

Define $f: A \to B$ and $g: B \to D$ by

$f: a \mapsto 5, b \mapsto 7, c \mapsto 12, d \mapsto 15, e \mapsto 18, f \mapsto 10$

$g: 3 \mapsto q, 5 \mapsto q, 7 \mapsto r, 10 \mapsto s, 12 \mapsto u, 15 \mapsto u, 18 \mapsto w$

i) Find the composition map $gof : A \to D$ using a diagram. (3 marks)

ii) Determine the inverse functions;

$f^{-1}: B \to A, g^{-1}: D \to B$ and $(gof)^{-1}: D \to A$ if they exist.

Explain. (3 marks)

(iii) Find all solutions to $(gof)(x) = u$. (2 marks)

(iv) Determine $g^{-1}(E)$, where $E = \{p,r,u\}$ (2 marks)

QUESTION 3 (20 marks)

a) Explain what is meant by a **commutative** and an **associative** binary operation.

(2 marks)

b) Which of the following binary operations defined on $\Re$ (the set of real numbers) is commutative or associative.

i) $x * y = xy + x + y$ (4 marks)

ii) $x * y = x^2 + 3y$ (4 marks)

c) Differentiate between an **identity** and **zero element** for a binary operation $*$ defined on a set A. (2 marks)

d) Determine the identity and zero elements if any for the binary operation in b(i) above. (4 marks)

e) An element is said to be **idempotent** under a binary operation $*$ if $b * b = b$. Determine all idempotent elements in $\langle \mathbf{Z}_{10}, \bullet \rangle$ by first making a table for x^2 for all x in $\mathbf{Z}_{10}$. Here $\bullet$ is multiplication modulo 10. (4 marks)

QUESTION 4 (20 marks)

The tables below show the Cayley table of a ring $\langle R, +, \bullet \rangle$

+	*k*	*l*	*m*	*n*	*p*	*q*	*r*
k	*p*	*k*	*n*	*r*	*q*	*m*	*l*
L	*k*	*l*	*m*	*n*	*p*	*q*	*r*
m	*n*	*m*	*k*	*p*	*r*	*l*	*q*
n	*r*	*n*	*p*	*q*	*l*	*k*	*m*
p	*q*	*p*	*r*	*l*	*m*	*n*	*k*
q	*m*	*q*	*l*	*k*	*n*	*r*	*p*
r	*l*	*r*	*q*	*m*	*k*	*p*	*n*

•	*k*	*l*	*m*	*n*	*p*	*q*	*r*
k	*q*	*l*	*n*	*k*	*r*	*p*	*m*
l	*l*	*l*	*l*	*l*	*l*	*l*	*l*
m	*n*	*l*	*r*	*m*	*q*	*k*	*p*
n	*k*	*l*	*m*	*n*	*p*	*q*	*r*
p	*r*	*l*	*q*	*p*	*n*	*m*	*k*
q	*p*	*l*	*k*	*q*	*m*	*r*	*n*
r	*m*	*l*	*p*	*r*	*k*	*n*	*q*

(a) Determine the additive and multiplicative identities. (2 marks)

(b) Find the additive inverse of each element. (3 marks)

(c) Determine the multiplicative inverse of each element. (3 marks)