KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

OPEN, DISTANCE AND E-LEARNING EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

<u>SMA 230: VECTOR ANALYSIS</u>

DATE: **Friday 4th February 2011** **TIME:** **8.00a.m -10.00a.m**

INSTRUCTIONS

Answer question <u>ONE</u> and Any other <u>TWO</u> questions

Question One

a) Given three vectors $\underset{\sim}{A} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{B} = 2\underset{\sim}{i} - 4\underset{\sim}{j} - 3\underset{\sim}{k}$, $\underset{\sim}{C} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$

i) The magnitude of $2\underset{\sim}{A} - 3\underset{\sim}{B} - 5\underset{\sim}{C}$.

ii) The volume of the parallel piped whose edges are represented by $\underset{\sim}{A}, \underset{\sim}{B}, \textit{and } \underset{\sim}{C}$. [5marks]

b) The position vector of a particle in space at time t is given by

$r(t) = (3t+1)\underset{\sim}{i} + \sqrt{3t}\,\underset{\sim}{j} + t^2\,\underset{\sim}{k}$

Find the angle between the velocity and the acceleration vectors at time t = 0.

[4marks]

c) If $\underset{\sim}{A} = 2\,yz\,\underset{\sim}{i} - x^2y\,\underset{\sim}{j} + xz^2\,\underset{\sim}{k}$ *and* $\phi = 2x^2yz^3$ find the following at the point (1, -1, 1)

i) $\nabla\phi$

ii) $\nabla.\underset{\sim}{A}$

iii) $\nabla X\,\underset{\sim}{A}$ [5marks]

d) Evaluate the double integral

$$\int_1^2 \int_1^{x^2} \left(x^2 + y^2\right) dy\, dx$$ [4marks]

e) Find a unit normal to the surface

$2x^2 + 4yz - 5z^2 = -10$ at the point P (3, -1, 2). [4marks]

f) If $\underset{\sim}{A} = \left(4xy - 3x^2 z^2\right)\underset{\sim}{j} + 2x^2 \underset{\sim}{l}\, 2x^3 z \underset{\sim}{k}$

Prove that $\int_C \underset{\sim}{A} . d\underset{\sim}{r}$ is independent of the curve C joining two given points and hence evaluate it. [5marks]

g) Use Green's theorem in the plane to evaluate

$$\oint_C \left(xy + y^2\right) dx + x dy$$

where C is the closed curve of the region $y = x$ and $y = x^2$.

Question Two

a) A particle travels so that its acceleration is given by

$\underset{\sim}{a} = 2e^{-t} \underset{\sim}{i} + 5 \cos t \underset{\sim}{j} - 3 \sin t \underset{\sim}{k}$.

If the particle is located at (1, -3, 2) at $t = 0$ and is moving with a velocity given by

$4\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$, find

i) The velocity

ii) The displacement at any time t. [7marks]

b) i) Show that

$\underset{\sim}{F}\left(2xy + z^3\right)\underset{\sim}{i} + x^2 \underset{\sim}{j} + 3xz^2 \underset{\sim}{k}$ is conservative force field.

ii) Find its potential and the work done in moving an object in this force field from (1, -2, 1) to (3, 1, 4). [8marks]

c) If $\underset{\sim}{A} = \left(3x^2 - 6yz\right)\underset{\sim}{i} + \left(2y + 3xz\right)\underset{\sim}{j} + \left(1 - 4xyz^2\right)\underset{\sim}{k}$

Evaluate $\int_C \underset{\sim}{A} . d\underset{\sim}{r}$ from (0, 0, 0,) to (1, 1, 1) along the path C given by

$x = t$, $y = t^2$, $z = t^3$. [5marks]

Question Three

a) State Green's theorem in the plane verify Greens theorem for the integral

$$\oint_c (x^2 + y^2)dx + (x + 2y)dy$$

Over the region bounded by the arc $x^2 + y^2 = 4$, $x = 0$, $y = 0$. [8marks]

b) Use the divergence theorem to evaluate

$\iint_s \bar{F}.Nds$, where $\underset{\sim}{F} = x^2 \underset{\sim}{i} + xy \underset{\sim}{j} + x^3y^3 \underset{\sim}{k}$ and S is the surface of the tetrahedron bounded by the plane $x + y + z = 1$ and the co-ordinate planes, with outward unit normal vector $\underset{\sim}{N}$

c) Evaluate using Stoke's theorem

$\oint_c xydx + xy^2dy$ where C is the square on the xy – plane with vertices (1, 0) , (-1, 0) , (0 ,1), (0,-1) [5marks]

Question Four

a) Show that

$\nabla r^n = nr^{n-2} \underset{\sim}{r}$ where $\underset{\sim}{r}$ has its usual meaning. [5marks]

b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) in the direction $2\underset{\sim}{i} - 3\underset{\sim}{j} + 6\underset{\sim}{k}$, [5marks]

c) i) State the Frenet-Serret formulae

ii) For the space curve

$$\underset{\sim}{r}(t) = 6\sin 2t \underset{\sim}{i} + 6\cos 2t \underset{\sim}{j} + 5t \underset{\sim}{k}$$

Determine $\underset{\sim}{T}$, $\underset{\sim}{N}$, $\underset{\sim}{B}$, k , *and* τ where the symbols have their usual meaning. [10marks]

Question Five

a) i) Determine the value of *m* such that $\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}$ below are coplanar.

$\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}, \; \underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + m\underset{\sim}{k} \quad c = \underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}.$ [3marks]

ii) Prove that the vector

$\underset{\sim}{A} = 3y^4z^2\underset{\sim}{i} + 4x^3z^2\underset{\sim}{j} - 3x^2y^2\underset{\sim}{k}$ is solenoidal. [3marks]

b) Show that the volume of the region bounded by the parabolic cylinder $z = 4 - x^2$ and the planes $x = 0$, $y = 0$, $y = 6$, $z = 0$ is given by the triple integral below and hence evaluate it.

$\int_0^2 \int_0^6 \int_0^{4-x^2} dz\, dydx$ [6marks]

c) If $\underset{\sim}{A} = 2x^2\underset{\sim}{l} - 3yz\underset{\sim}{j} + xz^2\underset{\sim}{k}$ and $\phi = 2z - x^3y$, find the following (1, -1, 1)

i) $\underset{\sim}{A}.\nabla\phi$

ii) $\underset{\sim}{A}\; X\; \nabla\phi$ [4marks]