KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND EDUCATION

SMA 231: DYNAMICS I

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DATE: MONDAY 2ND APRIL 2012 **TIME: 8.00 A.M. – 10.00 A.M.**

INSTRUCTIONS

Answer question ONE and any Two of the remaining questions.

Show all the necessary working.

QUESTION ONE (30 MARKS)

a) A particle of mass m move in the xy-plane so that its position vector is $\bar{r} = a \cos wt\ \bar{i} + b \sin wt\ \bar{j}$ where a , b and w are positive constants and $a > b$

i) Show that the particle moves in an ellipse

ii) Prove that the force acting on the particle is always directed towards the origin (4 marks)

b) Let us assume that there exists a scalar function V such that $\bar{F} = -\bar{\nabla} F$. Then prove that the total work done in moving the particle along curve C from P_1 to P_2 is

$W = V(P_1) - V(P_2)$ (4 marks)

c) Prove that the moment of force or torque about the origin O of a coordinate system is equal to the time rate of change of angular momentum (4 marks)

d) Prove that the range of the projectile is a maximum when the launching angle is 45^o. (4 marks)

e) State Newton's collision rule. Two marks m_1 and m_2 travelling in the same straight line collide. Find the velocities of the particles after collision in terms of the velocities before collision. (6 marks)

f) Determine unit vectors in cylindrical coordinates. Hence represent a vector

$$\bar{A} = z\bar{i} - 2x\bar{j} + y\bar{k}$$

in cylindrical coordinates and determine the components A_ρ , A_ϕ and A_z

(8 marks)

Question Two (20 marks)

Define a projectile. A projectile is launched with initial speed V_o at an angle α with the horizontal. Find

i) the position vector at any time t,
ii) the time to reach the highest point ,
iii) the maximum height reached,
iv) the time of flight back to earth,
v) the range, and
vi) show that the path of the projectile is a parabola (20 marks)

Question Three (20 marks)

a) A particle P of mass m slides without rolling down a frictionless inclined plane AB of angle α . If it starts from rest at the top A of the incline, find

i) the acceleration,
ii) the velocity and
iii) the distance traveled after time t

b) An object slides on a surface of ice along the horizontal straight line OA. At certain point in its path the speed is V_o and the object then comes to rest after traveling a distance x_o. Prove that the coefficient of friction is $V_o^2 / 2gx_o$, where g is the acceleration due to gravity. (10 marks)

Question Four (20 marks)

a) Define the terms amplitude, period and frequency of simple harmonic motion. (3 marks)

b) An object of 100N weight suspended from the end of a vertical spring of negligible mass stretches it 0.16 m. Determine the position of the object at any time if initially it is pulled down 0.05 m and then released. Find the frequency, period and the amplitude of the motion.
(Use g = 10 m / sec^2) (10 marks)

c) Prove that the force $\overline{F} = -Kx\bar{i}$ acting on a simple harmonic oscillator is conservative. Hence find the potential energy of the oscillator and express the principle of conservation of energy. (7 marks)

Question Five (20 marks)

a) Derive an expression for $\overline{\nabla}\phi$ in orthogonal curvilinear coordinates (8 marks)

b) Show that in orthogonal coordinates $\overline{\nabla} X (A_1 \overline{e_1}) = \frac{\overline{e_2}}{h_3 h_1}(A_1 h_1) - \frac{\overline{e_3}}{h_1 h_2}(A_1 h_1)$ (6 marks)

c) Express
Curl $\overline{A} = \overline{\nabla} X \overline{A}$ in orthogonal coordinates (6 marks)