KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

OPEN, DISTANCE AND E-LEARNING EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SMA 260: PROBABILITY AND STATISTICS II

DATE: **Tuesday 5th July 2011** **TIME:** **2.00p.m – 4.00p.m**

INSTRUCTIONS:

Answer question ONE and any other TWO questions

1. a) Define the probability distribution function of a continuous random variable X. [2marks]

   b) If X is a discrete random variable with probability distribution given by

   $$f(x) = \begin{cases} k, & x = 1,2,3,4 \\ 0 & otherwise \end{cases}$$

   i) Determine the value K . [2marks]

   ii) Find $\Pr(X \geq 3)$ [2marks]

   c) Suppose that a random variable X has a continuous distribution for which the p.d.f is as follows

   $$f(x) = \begin{cases} \frac{3}{2}(1 - x^2), & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

   Determine the median, mean and variance of this distribution. [11marks]

   d) Le X be a random variable which is normally distribution with mean 10 and variance 4. Compute

   i) $P(X < 8)$

ii) $P\ (8 < X < 12)$ [4marks]

e) A discrete random variable X has a probability generating function by $P\ (s) = q + pS$. Obtain the mean and variance of X. [4marks]

f) If X is a continuous random variable with p.d.f given by

$$F(x) = \begin{cases} \frac{1}{2}x & , \quad 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

Obtain the distribution function, $F(x)$, of X. [5marks]

2. a) Let Y be normally distributed random variable with mean μ and variance σ^2 . $Obtain$ the moment generating function of X. [7marks]

b) Let X be normally distributed variable with mean 4 and variance 16.

Find i) $P(|X| < 3)$ [3marks]

ii) $P\ (2 \leq -3X + 1 \leq 8)$ [4marks]

c) Let X have the following p.d.f

$$f\ (x) = \begin{cases} 4x^3 & o < x < 1 \\ 0 & otherwise \end{cases}$$

Find the p.d.f of a new random variable. Y = 3X -1 [6marks]

3. a) If the random X has a continuous distribution for which the p.d.f is given by

$$f\ (x) = \begin{cases} \frac{\Gamma(\alpha + \beta)}{\Gamma\alpha\ \Gamma\beta} & x^{\alpha-1} \quad (1-x)^{\beta-1}, \quad 0 < x < 1 \\ 0 & otherwise \end{cases}$$

Obtain the mean and variance of X. [8marks]

b) A random variable is defined to have Poisson distribution with p.d.f.

$$f\ (x) = \begin{cases} e^{\lambda}\ \frac{\lambda^x}{x!}\ , & x = 0, 1, 2, \ldots \\ 0 & otherwise \end{cases}$$

Obtain the moment generating function of X. Hence determine the mean and variance of X.

c) Use the Poisson approximation to compute the probability $P\ (X \leq 4)$ where X is binomial with $n = 120\ and\ p = 0.04$.

4 a) Let X be a continuous random variable with p.d.f given by

$$F(x) = \begin{cases} k(8 - x^2), & -2 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

i) Determine the value of the constant k. [2marks]

ii) Obtain the mean, mode and variance of X. [12marks]

b) Let Y be a discrete random variable with probability function given by

$$f(x) = \begin{cases} 0.2, & x = -1 \\ 0.3, & x = 0 \\ 0.1, & x = 1 \\ 0.4, & x = 2 \\ 0, & otherwise \end{cases}$$

Obtain the distribution function, $F(x)$, of X and sketch its graph.

[6marks]

5) a) If Y has a Poisson distribution with $P(X = 0) = P(X = 1)$, what is $P(X = 0 \; or \; 1)$. [15marks]

b) Let $\emptyset(t) = \ln(M(t)$, $where$ $M(t)$ is the moment-generating function of a distribution show that $\emptyset'(0) = \mu \; and \; \emptyset''(0) = \sigma^2$ where $\mu \; and \; \sigma$ are mean and variance of distribution. [6marks]

c) In an examination the average mark was 76.5 and the standard deviation was 9.5. If 15% of the class scored grade A and the marks are assumed to follow a normal distribution, what is the lowest possible grade A mark and the highest grade B mark? [9marks]

Table 2 CUMULATIVE NORMAL DISTRIBUTION

$$\Phi(x)=\int_{-\infty}^{x}\frac{1}{\sqrt{2\pi}}e^{-t^2/2}\,dt$$

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

x	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.891	4.417
$\Phi(x)$	.90	.95	.975	.99	.995	.999	.9995	.99995	.999995
$2[1-\Phi(x)]$	.20	.10	.05	.02	.01	.002	.001	.0001	.00001