KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 300: REAL ANALYSIS I

DATE: THURSDAY 25TH NOVEMBER 2010 **TIME:** 8.00 A.M – 10.00 A.M

INSTRUCTIONS:

Question One (30 marks)

(a) Let A and B be subsets of some universal set **U**. Prove that $(A \cap B)^c = A^c \cup B^c$ [4 marks]

(b) Let A = $\{x \in \Re : x \neq 3\}$ and define $f(x) = \dfrac{4x}{x-3}$ for all $x \in A$

Show that f is injective and determine a set B such that f: A $\to B$ surjective. [3 marks]

(c) State the completeness axion of the set $\Re$ of real numbers. [1 mark]

(d) Find the supremum, infimum maximal and minimal elements of the set

$S = \{1 + \frac{1}{n}, n \in N\}$ [2 marks]

(e) Prove that the intersection of any finite collection $(F_i)_{i=1}^{n}$ of open sets in $\Re$ is open. [5 marks]

(f) Justify the following statements

i) The empty set is both open and closed [2 marks]

ii) Every finite set is closed [2 marks]

(g) Prove that every convergent sequence is Cauchy sequence. [4 marks]

(h) Using a suitable Convergence test , determine whether or not the series

$\sum_{n=1}^{\infty} \dfrac{\log n}{2^n}$ Converges [4 marks]

(i) Let $f : \Re \to \Re$ be defined by $f(x) = 2x + 1$. Prove that f is continuous at $x = a$.

[3 marks]

Question 2 (20 marks)

(a) Show that the open interval (a,b) of $\Re$ is an open set. [3 marks]

(b) If A and B are non-empty subsets of $\Re$, prove that if B is closed and A is open then $A \setminus B$ is open and $B \setminus A$ is closed. [4 marks]

(c) The arbitrary union of a collection A_α ,$\alpha \in J$ of open sets in $\Re$ is open. Prove.

[5 marks]

(d) Prove that a finite subset $K\ of\ \Re$ is compact. [4 marks]

(e) Find (i) $Int(\ N\ )$ (ii) $Int\ Z$, (iii) Int $(Q)\ Int(\Re)$ [4 marks]

Question Three (20 marks)

(a) Define the terms

(i) Convergent sequence [2 marks]

(ii) Bounded sequence [2 marks]

(iii) Cauchy sequence [2marks]

(b) Show that that $\lim\limits_{n \to \infty} \dfrac{5n+4}{n+3} = 5$ [5 marks]

(Use the definition of convergence of a sequence) [5 marks]

(c) Prove that a sequence $\alpha_n\ in\ \Re$ has a unique limit. [5 marks]

(d) Every convergent sequence of real numbers bounded. Use a suitable example to show that the converse of this statement is not true. [4 marks]

Question Four (20 marks)

(a) i) Define the concept of absolute and conditional convergence of a series. [2marks]

ii) Test for absolute or conditional convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+1}$$ [2marks]

(b) Use the integral test to determine the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \frac{n}{n^2+5}$$ [4 marks]

(c) Investigate the convergence or divergence of the following series, justify your answer in each case. [3 marks]

i) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2n^2+1}$ ii) $\sum_{n=1}^{\infty} \left(\frac{n-1}{n^2}\right)^n$

iii) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$ iv) $\sum_{n=1}^{\infty} \frac{n!}{n^2}$ [12marks]

Question Five (20 marks)

(a) Let E be a subset of $\Re$, and suppose $f : E \to \Re$ is a function. If $c \in E$, state the definition of the continuity of f at a point c. [2marks]

(b) Prove that the function $f : \Re \to \Re$ defined by $f(u) = x^2 - 3x + 4$ is continuous at $x = 2$ [5 marks]

(c) Sketch the graph of the function

$$f(x) = \begin{cases} 2x-1, x \geq 2 \\ |x-3|, x < 2 \end{cases}$$

Hence or otherwise, determine whether f is continuous at $x = 2$ [6 marks]

(d) i) Define the concept of uniform continuity of the function in $f : S \to \Re$ where $S \subset \Re$. [2 marks]

ii) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0,1)$ [5 marks]

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