KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

MOMBASA CAMPUS

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 300: REAL ANALYSIS I

DATE: Tuesday 3rd April, 2012 **TIME:** 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS: ***Answer Question ONE and any other TWO.***

QUESTION ONE (30MARKS)

(a) Prove that for any two subsets A and B of some universal set $A - B = A \cap B^c$. (4mks)

(b) Show that the empty set is open. (3mks)

(c) Let $A = \{x \in \Re : x \neq 5\}$ and define *f by* $f(x) = \dfrac{2x}{x-5}$ $\forall x \in \Re$ .Show that *f* is 1-1 and determine a set *B* such that $f : A \to B$ is onto. (5mks)

(d) Let $\{x_n\}$ and $\{y_n\}$ be convergent sequences of real numbers. Show that $x_n + y_n \to x + y$. (3mks)

(e) Using a suitable example, show that boundedness of a sequence does not necessarily imply convergence. (2mks)

(f) Find $\underline{\lim}\, x_n$ and $\overline{\lim}\, x_n$ for the sequence $x_n = n^2 \sin^2\left(\dfrac{n\pi}{2}\right)$ and hence determine whether it is convergent or divergent. (3mks)

(g) Use the alternating series test to investigate the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$. (4mks)

(h) State the completeness axiom of the set $\Re$ of real numbers. (2mks)

(i) Justify the statement

(i) The interval (0,1) is open in $\Re$. (1mk)

(ii) The intersection of an arbitrary family of open sets is not open. (3mks)

QUESTION TWO (20MARKS)

(a) Prove that

(i) The sum of the first *n* squares of natural numbers is $\dfrac{n(n+1)(2n+1)}{6}$. (5mks)

(ii) $\sqrt{5}$ is irrational. (5mks)

(b) Consider a sequence $x_n = 3 + \frac{1}{n}$ and find the least positive integer N such that $|x_n - 3| < \varepsilon$ for all $n > N$ if $\varepsilon = 0.001$. (3mks)

(c) (i) When is a sequence said to be Cauchy? (2mks)

(iii) Prove that a Cauchy sequence with a convergent subsequence is convergent. (5mks)

QUESTION THREE (20MARKS)

(a) State the following convergence tests

(i) Comparison test. (2mks)

(ii) Root test. (2mks)

(iii) Ratio test. (2mks)

(b) Determine the convergence of each series using a suitable test.

(i) $\sum_{n=1}^{\infty} \frac{n!}{5^n}$. (4mks)

(ii) $\sum_{n=1}^{\infty} x_n = \begin{cases} \frac{1}{3^n}, n\ odd \\ \frac{1}{2^n}, n\ even \end{cases}$. (3mks)

(iii) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3 + 7}$. (3mks)

(iv) $\sum_{n=1}^{\infty} \frac{x^3}{x^4 + 1}$. (4mks)

QUESTION FOUR (20MARKS)

(a) Let $g : A \to B$ and $f : B \to C$ be surjective functions. Show that $f_0 g$ is also surjective. (4mks)

(b) Show that the product of two rational numbers is rational. (3mks)

(c) (i) Let $X \subseteq \Re$ and suppose $f : X \to \Re$ is a function. If $x_0 \in X$ state the definition of the continuity of f at x_0. (2mks)

(ii) Show that the function $f : \Re \to \Re$ defined by $f(x) = 4x^2$ is continuous. (4mks)

(d) (i) When is a set said to be countable? (2mk)

(ii) Show that the set of odd positive integers is countable. (5mks)

QUESTION FIVE (20MARKS)

(a) Distinguish between a minimal and an infimum to a subset of $\Re$. Hence find the minimal and infimum of the set $S = \left\{ 1 + \frac{1}{n}, n \in \square \right\}$. (4mks)

(b) Prove that the union of an arbitrary family of open sets in $\Re$ is open. (5mks)

(c) Show that a set $E \subseteq \Re$ is open if and only if E^c is closed. (6mks)

(d) A convergent sequence $\{x_n\}$ in $\Re$ has a unique limit. Prove. (5mks)

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