



KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2009/2010

INSTITUTIONAL BASED PROGRAMME

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 305: COMPLEX ANALYSIS I

DATE: THURSDAY, 2ND SEPTEMBER 2010

TIME: 2.00 P.M. - 4.00 P.M.

INSTRUCTIONS: Answer question ONE and any other TWO questions.

QUESTION ONE - Compulsory (30 marks)

(a) If $z = 1 + \sqrt{3}i$, find z^5 . (3 marks)

(b) Determine whether or not the function $f(z) = \begin{cases} \frac{z^2 + 9}{z - 3i} & \text{if } z \neq 3i \\ 2 + 3i & \text{if } z = 3i \end{cases}$ is continuous at $z = 3i$ (3 marks)

(c) Differentiate $f(z) = 2z^3 + z$ using first principles. (4 marks)

(d) Show that $f(z) = e^{-y}e^{ix}$ is analytic. (3 marks)

(e) Find the linear transformation that maps $z_1 = 1$, $z_2 = 0$, $z_3 = -1$ onto $w_1 = i$, $w_2 = \infty$, $w_3 = 1$ respectively. (3 marks)

(f) Evaluate $\int_c \bar{z} dz$ from $z = 0$ to $z = 4 + 2i$ and then from $z = 2i$ to $z = 4 + 2i$ (4 marks)

- (g) Find the residues of $f(z) = \frac{z}{(z-2)(z+2)^2}$ at all its poles. (3 marks)
- (h) Expand $f(z) = (z-3)\sin\frac{1}{z+5}$ into a Laurent Series at $z = -5$. (3 marks)
- (i) Evaluate $\oint_c \frac{dz}{z^2 - iz + 6}$ around the circle $c: |z - 2i| = 1$ (4 marks)

QUESTION TWO (20 marks)

- (a) Prove that $\cos 4\theta = 8\sin^4 \theta - 8\sin^2 \theta + 1$ (5 marks)
- (b) Find the third roots of $z = -1 + i$ (5 marks)
- (c) Express $\sin^5 \theta$ in multiple angles. (5 marks)
- (d) Show that the function
- $$f(z) = \begin{cases} \frac{z^2 + 2iz}{z^2 - iz + 6}, & \text{if } z \neq -2i \\ 4i, & z = -2i \end{cases}$$
- is not continuous at $z = -2i$ (5 marks)

QUESTION THREE (20 marks)

- (a) Prove that $f(z) = z^3 - 2z$ is harmonic. (5 marks)
- (b) Given that the function $u(x, y) = y^3 - 3x^2 y$ is harmonic, find $v(x, y)$ such that $f(z) = u + iv$ is analytic. (5 marks)
- (c) Show that $\sin z = \sin x \cosh y + i \cos x \sinh y$. (6 marks)
- (d) Find the principle value of $(-1)^i$ (3 marks)

QUESTION FOUR (20 marks)

- (a) Find the image of the unit square in the z plane given by a transformation

$$f(z) = Bz + c \text{ where}$$

$$B = 3 + 4i$$

$$c = 2 + 3i$$

(5 marks)

- (b) Find the fixed points of $f(z) = \frac{3z - 10}{z + 1}$

(5 marks)

- (c) Prove that $f(z) = \frac{1}{z}$ maps the line $X = K$ onto a circle.

(5 marks)

- (d) Evaluate the Integral $\int_{(0,3)}^{(2,4)} (2y + x^2)(3x - y)dy$ along the parabola

$$X = 2t, y = t^2 + 3.$$

QUESTION FIVE (20 marks)

- (a) State without proof

(i) Cauchy's Integral Theorem

(2 marks)

(ii) Cauchy's Residue Theorem

(3 marks)

- (b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent Series valid for

$$0 < |z+1| < 2$$

(5 marks)

- (c) Evaluate $\oint_c \frac{e^z}{(z-2)(z-4)} dz$ when

(i) $|z| = 5$

(6 marks)

(ii) $|z| = 3$

(4 marks)
