KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 305: COMPLEX ANALYSIS I

DATE: TUESDAY, 27TH MARCH 2012 **TIME: 8.00 A.M. – 10.00 A.M.**

INSTRUCTIONS: Answer question ONE and any other two.

Question One (30 marks)

a). Find the cube roots of $z = -2 + 2i$. (4 marks)

b). Evaluate $\lim\limits_{z \to 2i}\left(\dfrac{z^2+4}{2z^2+(3-4i)z-6i}\right)$ (4 marks)

c). Obtain the Laurent's series expansion of $f(z) = \dfrac{z+1}{z^2-3z+2}$ about the singular point $z = 1$. Use the series obtained to find the residue of $f(z)$ at $z = 1$ and hence evaluate $\int_C \dfrac{z+1}{z^2-3z+2}\,dz$ where $C : |z| = 1$. (6 marks)

d). Use Cauchy's theorem to evaluate the following integral $\int_c \dfrac{z+i}{z^3+9z}\,dz$

Where $C : |z+2-i| = 3$. (6 marks)

e). Use Cauchy's integral theorem to evaluate $\int_C \dfrac{\cos z}{2z^2-5z+2}\,dz$, Where $C : |z| = \dfrac{3}{4}$ (4 marks)

f). Let $v(x, y) = \sin x \cosh y$. Find $u(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is harmonic. Express $f(z)$ in terms of z. (6 marks)

Question two (20 marks)

a) Show that $\cos^{-1}(z) = -i\ln\left(z + \sqrt{z^2-1}\right)$. Hence find all solutions to the equation $\cos z = 2i$ (7 marks)

b) State without proof the Cauchy's residue theorem. Hence use it to evaluate the following integral

$$\int_C \frac{1}{z^2\left(z^2+z-2\right)}\,dz \quad |z| = 3$$

(7 marks)

c) Show that $|\cos z|^2 = \cos^2 x + \sinh^2 y$ (6 marks)

Question three (20 marks)

a) Show that a necessary condition for a function $f(z) = u(x,y) + iv(x,y)$ to be analytic in a region R is that the four partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$ exist and satisfy Cauchy Riemann conditions. (7 marks)

b) Show that $u(x,y) = \ln(x^2 + y^2)$ is harmonic. Find $v(x,y)$ such that $u(x,y) + iv(x,y)$ is harmonic. Express $f(z)$ in terms of z. (8 marks)

c) Determine whether $f(z) = e^{\bar{z}}$ is analytic. Justify your answer (5 marks)

Question Four (20 marks)

a) Show that if a function $f(z)$ is analytic inside and on a simple closed contour C, then $\int_C f(z)dz = 0$ (7 marks)

b) Find all values of $(-2)^i$. (4 marks)

c) Solve the equation $\ln(z^2 - 1) = \frac{i\pi}{2}$. (5 marks)

d) Use De-Moivres theorem to show that $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$ (4 marks)

Question Five (20 marks)

a) If $f(z)$ is analytic inside and on a simple closed curve C, and if z_0 is inside C, then $f'(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^2}dz$ .Show that the second derivative of $f(z)$ at z_0 is given by

$$f''(z_0) = \frac{2!}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^3}dz.$$ (8 marks)

b) Evaluate $\int_{3=2i}^{7+8i} (2x - y)dx + (3y + x)dy$ along the curve $x = 2t + 1$, $y = 3t - 1$ (6 marks)

c) Evaluate $\int_C \frac{1-2z}{z(z-1)(z-3)}dz$ where C: $|z| = 2$ (6 marks)

****