KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ARTS, BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SMA 330: NUMERICAL ANALYSIS I

DATE: Monday 22nd November, 2010 **TIME:** 2.00 p.m. – 4.00 p.m.

INSTRUCTIONS

Answer question ONE and any other TWO questions.

Question One

a) Distinguish clearly between the following terms as used in computation:

i) Absolute error and relative error.

ii) Inherent and induced numerical instability.

iii) Interpolation and extrapolation. [3 marks]

b) If f(x,y) = xy, show that

$$\max e_r \text{ in } x^* y^* = \left|\frac{\varepsilon}{x^*}\right| + \left|\frac{\eta}{y^*}\right|$$

Where x* and y* are the approximate numbers with errors ε and η respectively to the exact number x and y.

Hence or otherwise evaluate as accurately as possible xy if x = 2.53 ± 0.01 and y = 2.73 ± 0.02. [6 marks]

c) What is the absolute error in approximating ⅓ by 0.33333? If 0.33333 is the correct representation of a quantity x to the five decimal places shown, what is the range of possible values of x? [3 marks]

d) Use Newton Raphson method to estimate the root of $f(x) = e^{-x} - x = 0$ by employing an initial guess of $x_o = 0$ and stopping criterion

$|x_{n+1} - x_n| < 0.5 \times 10^{-5}$ [4 marks]

e) Construct a difference table for the data below and use it to compute f(1.3)

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	1.112	1.219	1.636	2.054	2.323	3.011

[5 marks]

f) Use Lagrange's formula to produce a fourth degree polynomial for the following (x, f(x)) values. Evaluate f(3) and f'(3).

x	0	1	2	4	5
f(x)	0	16	48	88	

[5 marks]

g) Use the three point (n = 2) Gaussian quadrature formulae to approximate the definite integral

$$I = \int_0^3 x\sqrt{1+x^2}\,dx$$ [4 marks]

Question Two

a) i) Find an approximate value of pi using Trapezoidal and Simpson's rules where

$$\pi = \int_0^1 \frac{4}{1+x^2}\,dx \text{ with } n = 6$$

ii) Using five digit arithmetic find the relative error in using each of the above approximations. [8 marks]

b) Construct the difference table for the sequence of values f(x) = (0, 0, 0, ε, 0, 0, 0) where ε is an error and show that:

i) the error spreads and increases in magnitude as the order of the difference increases.

ii) the error in each column has binomial coefficient. [5 marks]

c) The table below has one incorrect entry for f(x) locate and correct the wrong value

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
F(x)	1.000	1.029	1.060	1.099	1.152	1.225	1.342	1.455	1.624	1.837	2.100

[7 marks]

Question Three

a) If the first significant digit in an approximation number x* is K and the number is correct to n significant digits, then show that

$$\text{mar } e_r = \frac{1}{K10^{n-1}}$$ [4 marks]

b) Consider a table of values of $f(x) \equiv \ln x$ for $1 \leq x \leq 7$, with a spacing of h for the values of x. Bound the error of linear interpolation in this table. How small should h be chosen if the error in this interpolation is to be less than 5×10^{-6}? [4 marks]

c) Obtain the Taylor's approximation for $\ln(2 + h)$. Hence estimate ln 2.3026 correct to 3.dp. (Take $\ln 2 = 0.6931$). [4 marks]

d) Find the approximate value of the intergral $\int_0^1 \frac{1}{1+x} dx$ using composite trapezoidal rule with 2, 3, 5 and 9 nodes and Romberg integration. Compare the results with the exact value. [8 marks]

Question Four

a) i) Use lagrange interpolation process to obtain a polynomial of least degree that assumes these values:

x	0	2	3	4
y	7	11	28	63

[4 marks]

ii) Construct the divided difference table for the table in (i) above and find the Newton interpolating polynomial. Show that the polynomials obtained in are identical although their forms may differ. [6 marks]

b) i) Show that the equation $x^3 + 4x^2 - 10 = 0$ has a root in the interval [1, 2]. Perform five iterations of the bisection method to obtain a positive root to the equation correct to 4 d.p. [6 marks]

ii) Using Aitken's Δ^2 method, find the root α for

$f(x) = x^3 + 4x^2 - 10 = 0$ [4 marks]

Question Five

a) Given $f(x) = x^4 - 2x^3 + 3x^2 - x + 1$, the following values are calculated when x = -4(1) 5

437, 166, 47, 8, 1, 2, 11, 52, 173, 446

Form a backward difference table upto the 5th difference. Hence find f(3.12). [6 marks]

b) Using the Newton's forward difference prove that:

i) $hf'(x_o) = \Delta f_o - \frac{1}{2}\Delta^2 f_o + \frac{1}{3}\Delta^3 f_o - \frac{1}{4}\Delta^4 f_o + ...$

ii) $hf''(x_o) = \Delta^2 f_o - \Delta^3 f_o + \frac{11}{12}\Delta^4 f_o + ...$ [5 marks]

c) i) Derive the Newton Raphson method for solving f(x) = 0 and show that it converges quadratically for x_o if $f'(x_o) \neq 0$ [3 marks]

ii) Solve the quadratic equation $\partial x^2 - 6x - 3 = 0$. Using 4 digit arithmetic.

By taking an appropriate x_o use the Newton Raphson method to estimate the positive root of the equation above by performing 3 iterations. Evaluate the absolute error is any.

[6 marks]

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