KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

INSTITUTE OF OPEN, DISTANCE AND E-LEARNING

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SMA 330: NUMERICAL ANALYSIS I

DATE: Saturday 9th July, 2011 **TIME:** 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS

Answer question ONE and any other TWO questions.

Question one

a) Define the following terms as used in Numerical analysis

i) Inherent errors ii) Generated errors

iii) Numerical stability iv) Reformulation [4 marks]

b) Let p(x) = $((x^3 - 3x^2) + 3x) - 1$ and Q(x) = $((x-3)x+3)x - 1$. Use three digit rounding arithmetic to compute p(2.19) and Q(2.19).

Compare them with the true value p(2.19) = Q(2.19) = 1.685159. [5 marks]

c) Construct a difference table for the values shown in the table below and find the collocation polynomial in terms of x

x	0	1	2	3	4	5	6	7
f(x)	1	2	4	7	11	16	22	29

[5 marks]

d) Find the first and second derivatives of f(x) at x = 2 for the function f(x) = $\log_e x$ using the linear interpolation. Use the table of values below:

x	2.0	2.3	2.6
f(x)	0.69315	0.83291	0.95551

[5 marks]

e) Find the unique polynomial of degree 2 or less for the data below using the Lagrange interpolation formulae

x	0	1	3
f(x)	1	3	55

Find f(2.0) and verify that the sum of the coefficients is unity. [6 marks]

f) Evaluate $I = \int_0^1 \frac{1}{1+x} dx$ using Gauss-Legendre 3 point formula. [5 marks]

Question 2

a) Show that $E = 1 + \Delta$ and $E = (1 - \nabla)^{-1}$ where E is the shift operator. [6 marks]

b) For the following data, calculate the differences and obtain the forward and backward differences polynomials. Interpolate x = 0.25 and x = 0.35

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

[8 marks]

c) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines x = 0 and x = 1 and the curve through the point with the following co-ordinates

x	0.0	0.25	0.50	0.75	1.00
y	1.0000	0.9896	0.9589	0.9089	0.8415

Using Simpson's rule, estimate the volume of the solid formed giving your answer to 4 decimal places. [6 marks]

Question 3

a) Let x* and y* be two approximations to numbers with initial errors ε and n respectively to the exact numbers x and y. If f(x, y) = xy determine the maximum error in evaluating f.
Hence or otherwise evaluate as accurately as possible f given that
$x = 2.00 \pm 0.01$ and $y = 3.00 \pm 0.002$ [6 marks]

b) Apply the Newton-Raphson method to determine the root of the equation $f(x) = x^3 + x - 1$ such that $|f(x^k)| < 10^{-6}$ where x^k is the approximation to the root. Use $x_o = 1.0$. [6 marks]

c) A function f(x) is tabulated below

x	1.00	1.05	1.10	1.15	1.20
f(x)	1.0000	1.0247	1.0488	1.0725	1.0956

Estimate $f'(1.0)$ and $f''(1.0)$ and find the error in each case. [8 marks]

<u>Question 4</u>

a) Find the unique polynomial p(x) of degree 2 or less such that p(1) = 1, p(3) = 27, p(4) = 64 using

i) The Lagrange interpolation formulae.

ii) The Newton divided difference formula.Hence estimate P(1.5) [8 marks]

b) i) If a real number x is approximated by $\bar{x} = 456.123$ with relative error bound by 0.01, what are the end points of the interval of possible values of x? [4 marks]

ii) What is the absolute error in approximating ⅔ by 0.66666? If 0.66666 is the correct representation of a quantity x to five decimal places, what is the range of possible values of x? [2 marks]

c) Approximate $\int_0^4 2^x \, dx$ using composite trapezoidal rule with n = 1, 2 and 4. Apply Romberg integration to obtain a better approximation. [6 marks]

<u>Question 5</u>

a) Construct the difference table for the sequence of values

f(x) = (0, 0, 0, ε, 0, 0, 0) where ε is an error and show that:

i) the error spreads and increases in magnitude as the order of difference increases.

ii) the error in each column has binomial coefficient. [6 marks]

b) Use a difference table to find and correct a single error in the data below

1	3	11	31	69	113	223	351	521	739	1011

[6 marks]

c) i) Prove that $\sum_{k}^{n-1} \Delta y_k = y_n - y_o$ [4 marks]

ii) The Newton formula for the collocation polynomial can be written as

$$P_k = \sum_{i=0}^{n} \binom{k}{i} \Delta^i y_o$$

Prove that $y_3 = y_o + 3\Delta y_o + 3\Delta^2 y_o + \Delta^3 y_o$ [4 marks]

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