



KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ARTS, BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SMA 330: NUMERICAL ANALYSIS I

DATE: Thursday 5th April 2012

TIME: 11.00a.m – 1.00p.m

INSTRUCTIONS

Answer question ONE and any other TWO questions

Question One

a) Define the following terms as used in correction with errors:

- i) Relative error
- ii) Round-off error
- iii) Inherent error

[3marks]

b) Estimate the error in evaluating $f(x, y, z) = x^2 + y/z$ for $x = 1.23$, $y = 2.34$ and $z = 3.45$ where each of these is assumed to be correctly rounded to the number of significant figure shown. Hence find f .

[4marks]

c) Construct a difference table for the values shown in the table below and find collocation polynomial in terms of x find $f(4,5)$

x_k	0	1	2	3	4	5	6	7
y_k	1	2	4	7	11	16	22	29

[5marks]

d) Use Lagrange interpolation polynomial to calculate $f(2)$ from the following

x_i	0	1	3
$f(x_i)$	1	3	55

[4marks]

e) Set up a Newton Raphson interaction to estimate the square root of a given positive number C .

Hence obtain the square root of 2, starting with $x_0 = 1.4$ up to 3 iterations, correct to give decimal places. [4marks]

f) Two quantities x and y are found to be related as below

x	1	1.5	2.0	2.5	3.0	3.5	4.0
y	3.1	4.0	4.2	3.8	2.9	2.8	2.7

Assuming $y = f(x)$ is continuous find $\int_1^4 f(x) dx$ numerically. Using both Trapezoidal and Simpson's rules. [4marks]

g) Construct a divided difference table for the data below and use it to determine the interpolating polynomial. Hence find $P(4.2)$

x	0	1	2	4	6
$f(x)$	1	9	23	93	259

Question Two

a) State the difference between inherence and induced numerical instability [2marks]

b) i) Consider the sequence of values $f(x) = (0, 0, 0, \epsilon, 0, 0,)$ where ϵ is an error. Use a difference table to comment on the spread and magnitude of the error as the order of the difference increase. [4marks]

c) The table below has one incorrect entry for $f(x)$, locate and correct the wrong value.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$f(x)$	1.000	1.029	0.060	1.099	1.152	1.225	1.342	1.455	1.624	1.837	2.100

[2marks]

d) i) The Newton formula for the coloration polynomial can be written as

$$P_k = \text{EMBED Equation.3} \Delta^i y_0 \text{ prove that}$$

$$y_3 = y_0 + 3\Delta y_0 + 3\Delta^2 y_0 + \Delta^3 y_0 \quad [2\text{maks}]$$

ii) State the differential between interpolation and extrapolation and obtain the collocation polynomial for the data below using either Newton's backward interpolation formula. Hence interpolate at $x = 0.25$.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

[5marks]

Question Three

- a) Find $f'(2)$ and $f''(2)$ for the function $f(x) = \log x$ using linear and quadratic interpolation and the table of values below:

x	2.0	2.3	2.6
$f(x)$	0.69315	0.83291	0.9555

[5marks]

- b) Evaluate $I = \int_0^4 \cos x \, dx$

Using Gauss- Legendre formula

[3marks]

- c) Approximate $\int_0^4 2^x \, dx$ using composite trapezoidal rule with $n = 1, 2$ and 4 . Apply

Romberg integration to obtain a better approximation.

[7marks]

- d) Apply synthetic division to $P(x) = 2x^4 - 24x^3 + 100x^2 - 168x + 93$ to compute $P(2)$.

[3marks]

Question Four

- a) i) The value of π can be approximated using function

$$\pi = \int_0^1 \cos(x^2) \, dx$$

Approximate the value of π using Trapezoidal and Simpson's rules with $n = 6$

- ii) By using direct substitution obtain the value of π from the above function.

- iv) Using five digit arithmetic find the relative error in using each of the methods in (a) above.

[12marks]

- b) i) Show that the equation $f(x) = x^3 + x - 1$ has a root in the interval $[0,1]$.

Apply the Newton-Raphson method to determine the root of the equation starting with $x_0 = 1.0$ and performing 4 iterations.

[5marks]

- ii) Compute $\ln 9.2$ given that $\ln 9.0 = 2.1972$ and $\ln 9.5 = 2.2513$ by linear Lagrange interpolation.

[3marks]

Question Five

- a) Evaluate as accurately as possible x/y is $x = -2.73 + 0.2$ and $y = 4.63 + 0.01$ [4marks]
- b) Given the equation $x^3 + 4x^2 - 10 = 0$
- Show that the equation has a unique root in the interval $[1, 2]$
 - Establish that $F(x) = \left(\frac{10}{x} - 4x\right)^{\frac{1}{2}}$ is an iteration formula for the equation and test for its convergence in the neighbourhood of the unique root in (i) above. [6marks]
- c) i) Define the term truncation error. [1mark]
- ii) For what values of x can we replace $\sin x$ by $x - x^3/3$, with an error of magnitude not greater than 3×10^{-4} ? [4marks]
- d) Evaluate the integral $I = \int_{-1}^1 (1 - x^3)^{3/2} \cos x \, dx$ using Gauss – Legendre free point formula. [5marks]