# KENYATTA UNIVERSITY

# UNIVERSITY EXAMINATIONS 2011/2012

### SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

### SMA 334: FLUID MECHANICS II

**DATE**: Wednesday 4th April, 2012 **TIME:** 11.00 a.m. – 1.00 p.m.

---

**INSTRUCTIONS TO CANDIDATES**

*Answer Question ONE and any TWO other questions.*

Q1. (a) Suppose that there is a line source of strength m at $z = z_1$ where $\mathrm{Re}(z_1) \geq 0$. Show that no fluid crosses the plane $x = 0$ [3 marks]

(b) A vortex flow is given by $w = \frac{ik}{2\pi} \ln z$ where k is a real constant.

(i) Find the equations of streamlines and equipotentials.

(ii) Find the circulation $\Gamma$ when the curve c surrounds the origin 0. [7 marks]

(c) If $f(t) = U(t) + i\,V(t)$, where t is real. Find $f(\bar{z})$ and $\bar{f}(\bar{z})$. [3 marks]

(d) State and prove Milne-Thomson circle theorem. [7 marks]

(e) Find the complex velocity potential due to a uniform stream past a circular cylinder at incidence $\alpha$ to OX in terms of z. [4 marks]

(f) Discuss the flow due to a uniform line doublet at 0 its axis being along OX.

[6 marks]

Q2. (a) Find the complex velocity potential due to a line doublet parallel to the axis of a right circular cylinder. [5 marks]

(b) A vortex of circulation $2\pi k$ is at rest at the point $z = na$ $(n > 1)$, in the presence of a plane circular boundary $|z| = a$, around which there is a circulation $2\pi\lambda k$. Show that

$\lambda = 1/(n^2 - 1)$ [15 marks]

Q3. (a) Given the complex velocity potential $w = f(z) = \phi + i\psi$, derive the Cauchy-Riemann equations. [5 marks]

(b) Derive the expression for the complex velocity potential at P for a line doublet of strength $\mu$ per unit length at A, $z = z_1$, whose axis makes an angle $\alpha$ with the axis. [9 marks]

(c) Given an irrotational flow with velocity potential $\phi$, with velocity $q = [u, v) = -\nabla\phi$, and stream function $\psi$ such that $-u = \frac{\partial \psi}{\partial y}$, $v = \frac{\partial \psi}{\partial x}$. The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(i) Find equations relating stream function to velocity potential.

(ii) Show that velocity potential and stream function satisfy the laplace equation.

(iii) Obtain an equation to show that curves $\phi$ = constant (equipotentials) and $\psi$ = constant (streamlines) intersect orthogonally. [6 marks]

Q4. Derive the Navier-Stokes equation for the viscous flow in tubes of uniform cross-section, the velocity being $q = w(x, y)k$. Hence find the expression for velocity $w(x,y)$ for a tube having uniform elliptic cross-section. [Assume the translational equations]. [20 marks]

Q5. State the Theorem of Blasius and derive expressions for the force components [X,Y] and couple M on the cylinder due to the action of the fluid. [20 marks]

........................