KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ENGINEERING)

EMM 208: MATHEMATICS FOR ENGINEERING IV

DATE: THURSDAY, 12TH APRIL 2012 **TIME:** 11.00 A.M -1.00 P.M

INSTRUCTIONS

Answer question ONE and any other TWO

QUESTION ONE(30 MARKS)

1. a. Given the function $f(x)$ as $\begin{cases} 0 & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$

and $f(x) = f(x + 2\pi)$

i) State the period of the function [1 mark]

ii) Sketch the graph of the function [2 marks]

iii) Determine the Fourier co-efficient [4 marks]

iv) State the Fourier Series of the function [3 marks]

b. Solve the Boundary Value Problem $t\dfrac{\partial^2 u}{\partial x \partial t} + 2\dfrac{\partial u}{\partial x} = x^2$ $u(x,0) = 2, u(0,t) = 1$

[4 marks]

c. Find the Laplace transforms of $f(t) = e^{2t} + t^2$ [4 marks]

d. Using the methods of Separation of Variables solve the d.e

$\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ [6 marks]

e. Determine the Inverse Laplace transform of

$$F(S)=\frac{S}{S^2+4S+13}$$ [6 marks]

QUESTION TWO (20 MARKS)

2. a. State Fourier Series of a function $f(x)$ defined over the interval $-L<x<L$

[4marks]

b. Given the $f(x)=x^2$ for $0<x<2\pi$. Use the Series stated in 2(a) above to determine the Fourier Series of $f(x)$ and sketch its graph. [10 marks]

c. Differentiate between an odd and an even function. State the properties of each function. [6 marks]

QUESTION THREE(20 MARKS)

a. (i) Define a Laplace transforms [2 marks]

(ii) Find the inverse Laplace transforms of

$$F(S)=\frac{S+4}{S(s-1)(S^2+4)}$$ [6 marks]

b. Using the Laplace transform , find the solution of the initial value problem

$y''-4y'+4y=64\sin 2t$ where

$y(0)=0$ and $y'(0)=1$

c. Evaluate $\int_0^{\infty} te^{-3t}\sin t dt$ [4 marks]

QUESTION FOUR (20 MARKS)

4 a. (i) State the difference of the terms general solution, particular solution and singular solution as used in partial differential Equation. [3 marks]

(ii) Classify the following pde's

$$x\frac{\partial^2 u}{\partial x^2}+y\frac{\partial^2 u}{\partial x^2}+3y^2\frac{\partial u}{dx}=0$$

b. Show that $u(x,t)=e^{-8t}\sin 2x$ is a solution to the BVP $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$

Where $u(o,t)=u(\pi,t)=0$

$u(x,0)=\sin 2x$ [5 marks]

c. Solve the following pde's

(i) $\dfrac{d^2 z}{\partial x \partial y} = x^2 y \quad z(x,0) = x^2$ and $z(1, y)$ [4 marks]

(ii) $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u \quad and\ u = e^{-5y}$ when $x=0$ [5 marks]

QUESTION FIVE (20 MARKS)

5a. Solve the differential equation

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x \text{ where } y(0) = 0 \text{ and } y'(0) = 1$$ [7 marks]

b. A resistance R in Series with inductance L is connected with e.m.f E. The current t is given by

$$L\frac{di}{dt} + Ri = E(t).$$

If the switch is connected at t=0 and disconnected at $t = a$, find the current i in terms of t. [8 marks]

c. Solve the differential equation

$$\frac{d^2 y}{dx^2} + y = 0 \quad \text{Where } y = 1 \text{ and } {dy}/{dx} = -1 \quad \text{at } x=0$$ [5 marks]

Laplace Transform Table

	$f(t)$	$f(s) = L[f(t)]$
1.	e^{at}	$\frac{1}{s-a}$
2.	t^n	$\frac{n!}{s^{n+1}}$
3.	$sin\ at$	$\frac{a}{s^2+a^2}$
4.	$cos\ at$	$\frac{s}{s^2+a^2}$
5.	$e^{bt} sin\ at$	$\frac{a}{(s-b)^2+a^2}$
6.	$e^{bt}\ cos\ at$	$\frac{s}{(s-b)^2+a^2}$
7.	$t\ cos\ at$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
8.	$t^n\ e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
9.	$Sin\ h\ at$	$\frac{a}{s^2-a^2}$
10.	$cos\ h\ at$	$\frac{s}{s^2-a^2}$
11.	$\frac{d^n f(t)}{dt^n}$	$S^n f(s) - S^{n-1} f(o) - S^{n-2} f'(o) - S^{n-3} f''(o)$