KENYATTA UNIVERSITY
UNIVERSITY EXAMINATIONS 2011/2012
MOMBASA CAMPUS

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION, BACHELOR OF SCIENCE AND BACHELOR OF ARTS
SMA 335: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: Friday 30th March, 2012 **TIME:** 4.30 p.m. – 6.30 p.m.

INSTRUCTIONS: *Answer Question ONE and any other TWO questions.*

QUESTION ONE (COMPULSORY – 30 MARKS)

a) Find the particular solution of the differential equation $\frac{dy}{dx}+4x=y$ when $y(0)=4$. **4 mks**

b) Show that $f(x)=e^x+2x^2+6x+7$ is a solution of the differential equation **3 mks**

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=4x^2$$

c) Solve the equation $3\frac{d^2y}{dx^2}+4\frac{dy}{dx}+9y=0$ **4 mks**

d) Solve $(x^2-3y^2)dx+2xydy=0$ **4 mks**

e) Find a suitable integrating factor that makes the equation $(2x^2+y)dx+(x^2y-x)dy=0$ to be exact and hence solve the equation. **4 mks**

f) Let $\frac{dy}{dx}=\frac{y^2-2x^2-6xy}{3x^2-2xy+y^2}$.

i. Show that the equation is exact. **3 mks**

ii. Hence or otherwise, solve the equation. **2 mks.**

g) The population x of a certain city satisfies the law $\frac{dy}{dx}=\frac{1}{100}x$ where time t is measured in years. Given that the population of this city is 200,000 in 1980,

i. Find the equation connecting the population $x$and time t (>1980) **(3mks)**

ii. Determine in what year does the population double **(3mks)**

QUESTION TWO (20 MARKS)

a) Consider the equation $x^3 \frac{d^3y}{dx^3} - 4x^2 \frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 4\ln x$ .Use a suitable substitution to convert the equation into an ordinary differential equation with constant coefficients. And solve the equation **13 mks**

b) Find the differential equation representing all circles of radius 5 **7mks**

QUESTION THREE (20 MARKS)

a) Given the $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$.

i. Assume a solution of the form $y = e^{mx}$, find a solution of the resulting auxiliary equation. **2 mks**

ii. Find the complementary solution **1 mks**

iii. Use the method of undetermined coefficients to find a particular solution. **10 mks**

b) Find the power series solution near $x = 0$ for the differential equation $\frac{dy}{dx} - y = x^2$ **7mks**

QUESTION FOUR (20 MARKS

a) Use the method of variation of parameters to solve the differential equation $\frac{d^2y}{dx^2} + y = \csc x$ subject to the conditions $y(0) = y(\frac{\pi}{2}) = 0$ **8 mks**

b) The rate at which radioactive nuclei decay is proportional to the number of nuclei that are present in a given sample. Half of the original number of radioactive nuclei have undergone disintegration in a period of 50 years.

i. What percentage of the original radioactive nuclei will remain after 150 years? **8mks**

ii. In how many years will 75% of the original number remain? **4mks**