KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

EMM 304: MATHEMATICS FOR ENGINEERS V

DATE: Wednesday 11th April, 2012 **TIME:** 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS

Answer Question ONE and ANY OTHER TWO.

Question One – (30 marks)

(a) Determine the roots of the equation $x^3 - 3x - 5 = 0$ correct to three decimal places. Use Newton-Raphson method. [6 marks]

(b) Given the table below, determine the value of f(1.6) using Newton's forward formula

x	1	1.4	1.8	2.2
f(x)	3.49	4.82	5.96	6.5

[6 marks]

(c) Use forward and backward difference Newton's formula to find dy/dx and $\frac{d^2y}{dx^2}$ at the

(i) $x = 1.1$

(ii) $x = 1.6$

Given that

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

[8 marks]

(d) Apply Eulers modified method to solve the d.e $dy/dx = x + 3y$ subject to $y(o) = 1$ and hence approximate the value of y when $x = 1$. [5 marks]

(e) Using Runge-Kutta method of 3rd order, solve the differential equation

$dy/dx = x - y$ where $y(1) = 1$, and find $y(0.1)$

Use $y_1 = yo + \frac{1}{6}(k_1 + 4k_2 + k_3)$ where

$K_1 = hf(x_o, y_o)$

$K_2 = hf(x_o + \frac{h}{2}, y_o + \frac{k_1}{2})$

$K_3 = hf(x_o + h, y_o + k_2)$

[5 marks]

Question Two (20 marks)

(a) The following data gives corresponding values of pressure and specific volume of a super-heated steam

V	2	4	6	8	10
P	105	42.7	25.3	16.7	13

Estimate the value of $\int_2^{10} Pdv$ using

(i) Trapezoidal rule [5 marks]

(ii) Simpson's rule [5 marks]

(b) Given that $y = \ln(x + 1)$ use Taylor's Series to expand the function near $x = 1$ up to x^5. [5 marks]

(c) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using Simpson's rule [5 marks]

Question Three (20 marks)

(a) Use Newton-Raphson method to find the root of the equation given below correct to 3 decimal places.

$x^5 - 5x^2 + 3 = 0$ [5 marks]

(b) The bacteria concentration in a reservoir varies as $C = 4e^{-2t} + e^{-0.1t}$. Calculate the time required for the bacteria concentration to be 0.5 using Newton-Raphson iterative method. [7 marks]

(c) The table below shows the distance in miles of the visible horizon for the given heights in metre above the earth's surface.

x = height (metres)	100	150	200	250	300	350	400
y = distance (metres)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the value of y when

(i) $x = 218$ metres [4 marks]

(ii) $x = 410$ metres [4 marks]

Question Four – (20 marks)

(a) Using Newton's forward and backward difference formula find

$\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at

(i) $x = 1.00$ [4 marks]

(ii) $x = 1.25$ [4 marks]

(iii) $x = 1.15$ [4 marks]

given the table below:-

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.0000	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

(b) Obtain the solution of the equation $\frac{dy}{dx} = x + \sqrt{y}$ with the initial conditions $y = 1$ at $x = 0$ for the range $0 \leq x \leq 0.6$ in steps of 0.2 using Euler's modified method. [8 marks]

Question Five – (20 marks)

(a) Find the values of y at $x = 0.1$ and $x = 0.2$ to 5 decimal places of the d.e $\frac{dy}{dx} = x^2y - 1$ for $y(o) = 1$ using Taylor's Series method. [7 marks]

(b) Runge-Kutta's fourth order method is given by the formula

$y_1 = y_o + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where

$K_1 = hf(x_o, y_o)$

$K_2 = hf(x_o + \frac{1}{2}h, y_o + \frac{1}{2}k_1)$

$K_3 = hf(x_o + \frac{1}{2}h, y_o + \frac{1}{2}k_2)$

$K_4 = hf(x_o + h, y_o + k_3)$

Use the above formula to find an approximate value of y when $x = 0.2$ given that

$\frac{dy}{dx} = x + y$ and $y(o) = 1$

Use $h = 0.1$. [13 marks]

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