# KENYATTA UNIVERSITY

## UNIVERSITY EXAMINATIONS 2009/2010

## SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

## SMA 336: ORDINARY DIFFERENTIAL EQUATIONS II

**DATE: Tuesday 6th April, 2010** **TIME: 8.00 a.m – 10.00 a.m**

**INSTRUCTIONS: Answer Question ONE and any other TWO questions.**

QUESTION ONE

(a) i) Define the Wronskian of the $n^{th}$ order differential equations. [ 2 marks]

ii) Use the Wronskian to determine if the two function $e^{-x}\sin x$ *and* $e^{-x}\cos x$. Can yield a general solution of a differential equation. [ 3 marks]

(b) Determine the singular points in the differential equation.

$x(x^2-4)^2\, y'' \;+\; (x+2)\, y' \;+\; 3y = 0$ and classify them as regular or irregular.

[ 4 marks]

(c) Prove that the total differential equation below is integrable and hence solve it.

$yz^2dx - xz^2dy - (2xyz + x^2)dz = 0$. ( 5 marks)

(d) Define the Legendre polynomial $P_n(x)$ and hence or otherwise show that

$P_n(-x) = (-1)\ P_n(x)$. (4 marks)

(e) Find the family of solutions of the non-linear differential equations.

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} = 0$$

(f) Solve in terms of the Bessel function, the Bessel equation

$x^2y'' + xy'\left(x^2 - \frac{9}{4}\right)y = 0$ (4 marks)

Express the Bessel functions in the terms of the elementary functions

$\cos x$ *and* $\sin$ $x$ *given* that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \ \sin x$. (6 marks)

(g) Reduce the given initial value problem to an equivalent initial value problem for a first order system and write the matrix form. $y''' + 2y'' - y' - 2y = 0$. (2 marks)

QUESTION TWO

(a) Use an appropriate power series method to solve the differential equations.

$3xy'' + (2 - x)y' - y = 0$ ( 8 marks)

(b) Solve the following equation and determine the constants of integration when the initial conditions are as stated.

$y\,y'' + 4y^2 - \frac{1}{2}(y')^2 = 0.$ , $y(0) = 1$ , $y^1(0) = \sqrt{8}$. ( 6 marks)

( c) Prove the following

i) $\int_{-1}^{1} P_n(x) P_m(x)\,dx = 0, \quad n \neq m$. (3 marks)

ii) $J_3(x) + 3J_0^1(x) + 4J_0'''(x) = 0$. (3 marks)

QUESTION THREE

(a) i) State and prove the necessary condition for a total differential equations. $P\,dx + Q\,dy + R\,dz = 0$ to be integrable. (5 marks)

ii) Determine $3x^2 dx + f(y)dy - (x^3 + y^3 + e^{2z})\,dz$ is integrable and hence. Solve it. (5 marks)

(b) Test the differential equations below for both integrability and exactness and hence. Solve it if possible. $2xz(y - z)dx - z(x + 2z)dy + y(x^2 + 2y)dz = 0$ (5 marks)

(c ) Show that $y = x$, is a solution of $(x^2 + 1)y'' - 2xy' + 2y = 0$ and find a linearly independent solution by reducing the order. (5 marks)

QUESTION FOUR

(a) Find the general solution of the system

$\dot{x}_1 = 3x_1 + 2x_2 + 2x_3$

$\ddot{x}_2 = x_1 + 4x_2 + x_3$

$\ddot{x}_3 = -2x_1 - 4x_2 - x_3$ (6 marks)

(b) Find the power series solutions in the power of $(x - 1)$ of the initial – value problem.

$xy'' + y' + 2y = 0$

$y(1) = 2$ and $y'(1) = 0$ ( 10 marks)

( c) Find a polynomial solution to the legendre equation

$(1 - x^2)\,y'' - 2xy' + 20y = 0$ ( series solution not required). (4 marks)

QUESTION FIVE

(a) i) State the Existence and uniqueness Theorem for an $n^{th}$ order linear equation.

( 2 marks)

ii) Prove that the initial value problem $x^2 y''' + 3xy'' - 8y' = e^x$,

$y(2) = 2, \quad y'(2) = 3, \quad y''(2) = -1$ *does* have a solution. (4 marks)

(b) Show that the functions $y_1(x) = e^{-x/2}\sin\left(\sqrt{\frac{3}{2}}\right)x$ and $y(x) = e^{-x/2}\cos\left(\sqrt{\frac{3}{2}}\right)x$ are linearly independent and find the differential equation that they solve. (6 marks)

(c) Find the equation of the curve whose radius of curvature is double radius of curvature is double the normal and opposite directed. (8 marks)

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