KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

INSTITUTE OF OPEN LEARNING

EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SMA 336: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: FRIDAY, 4TH FEBRUARY 2011 **TIME: 11.00 A.M. - 1.00 P.M.**

INSTRUCTIONS: Answer Question ONE and any other TWO questions.

Question One

a) i) Define the term Wronskian of n functions.

ii) Use the Wronskian test whether the following are linearly independent or dependent e^{3x} and e^{2x}.

iii) Hence write down the differential equation whose solutions are given in (ii) above.

(6 marks)

b) Find the general solution to the system of equations

$$dx/dt = 2x + 3y$$

$$dy/dt = 2x + y$$

(4 marks)

c) Show that the total differential equation below is both exact and integrable and hence solve it.

$$(x - 3y - z)dx + (2y - 3x)dy + (z - x)dz = 0$$

(5 marks)

d) Find the polynomial to the Legendre equation

$$(1 - x^2)y'' - 2xy' + 20y = 0$$

(series solution not required) (4 marks)

e) i) Define Bessel differential equation and Bessel function.

ii) Show that $J_{-n}(x) = (-1)^n J_n(x)$.

(5 marks)

f) i) State the Existence and Uniqueness Theorem for a second order linear equation.

ii) Show that there exists a unique solution for the equation.

$\cos x \, d^2y/dx^2 + x^2 \, dy/dx - xy = \sin x$ where $x \neq \pi/2$ (5 marks)

Question 2

a) i) State and prove the condition of integrability for the total differential equation

$pdx + Qdy + Rdz = 0$ (6 marks)

b) Test the integrability of the total differential equations below and solve where possible.

i) $(y+z)dx - (x+z)dy + (2y - x + z)dz = 0$

ii) $(e^y - ze^x)dx + xe^y dy - e^x dz = 0$

iii) $(y^2 z + yz^2)dx + zx(z+x)dy + xy(x+y)dz = 0$

(14 marks)

Question 3

a) Write down the second order homogeneous differential equation below in normalized form

$P_0(x) \, d^2y/dx^2 + P_1(x) \, dy/dx + P_2(x)y = 0$ (2 marks)

b) Use the above to define the following

i) an ordinary point

ii) a singular point

iii) a regular singular point

iv) an irregular singular point

(4 marks)

c) Determine the singular points of the following differential equation and classify them as regular or irregular

$$\left(x^2-1\right)\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+xy=0$$ (6 marks)

d) Show that $y=x$ is a solution of the differential equation

$$x^2y'''-3x^2y''+x\left(6-x^2\right)y'-\left(6-x^2\right)y=0$$

By reducing the order find the complete solution of the d.e. (8 marks)

Question 4

a) i) Define the term non-linear differential equation. (2 marks)

ii) Show that the equation below is non-linear

$\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3y=0$ is non-linear. Hence solve the equation

given the initial conditions $y(0)=1,\ \ y'(0)=-1$. (8 marks)

b) i) Show that $\left(x^pJ_p(x)\right)'=x^pJ_{p-1}(x)$ (3 marks)

ii) Find the two particular solutions to the Bessel equation

$x^2y''+xy'+\left(x^2-\frac{9}{4}\right)y=0$ in terms of elementary functions

$\cos x$ and $\sin x$ given that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x.$ (7 marks)

Question 5

a) Solve the following differential equation in a series of powers of x

$$y'' + y = 0$$

given that $y(0) = 1$ and $y'(0) = 5$ (10 marks)

b) Find the general solution of the first order system

$$dx/dt = 3x + 2y + 2z$$
$$dy/dt = x + 4y + z$$
$$dz/dt = -2x - 4y - z$$

(10 marks)

****