KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

INSTITUTION BASED PROGRAMME

AUGUST SESSION EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

SMA 336: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: Monday 2nd January, 2012 **TIME:** 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS*: Answer question ONE and any other TWO questions.*

Question one

a) i) Define the Wronskian of a set of functions $\{f_1, f_2, f_3 \ldots f_n)$ as used in linear differential equations. [1 mark]

ii) Use the Wronskian to show that $\{1, e^x, e^{2x}\}$ form a fundamental set of solutions. Hence find the differential equation whose solutions are 1, e^x, e^{2x}. [4 marks]

b) Find the general solution of the following system of first order equations

$$\frac{dx}{dt} = x + y$$

$$dy/dt = 5x - 3y$$ [4 marks]

c) Show that the total differential equation

$(yz + z^2)dx - xzdy + xydz = 0$ is integrable and hence solve it. [5 marks]

d) Determine the singular points in the differential equation below and classify them as regular or irregular

$$x(x^2 - 4)y'' + (x + 2)y' + 3y = 0$$ [4 marks]

e) i) Find the polynomial solution to the Legendre equation

$$(1 - x^2)y'' - 2xy' + 12y = 0$$ [4 marks]

(12y)

ii) Prove that $P'_{(n+1)} - xP'_{(n)} = (x + 1)P_n$ where P_n represents the Legendre polynomial of order n. [3 marks]

f) Show that $\frac{dy}{dx} = 2x^2 + y^2,\ y(0) = 3$

has a unique solution in R^2. [2 marks]

g) Find the recurrence relation or recursion formula for the coefficient of the general power series near $x_o = 0$ of $y'' - 4y = 0$ [3 marks]

Question two

a) State and prove the integrability test for the total differential equation

$Pdx + Qdy + Rdz = 0$ [5 marks]

b) Test for the integrability and exactness of the total differential equations below and hence solve where possible

i) $(y+z)dx - (x+z)dy + (2y - x + z)dz = 0$

ii) $(3x^2y^2 - e^x z)dx + (2x^3y + \sin z)dy + (y\cos z - e^x) = 0$ [7 marks]

c) i) Explain the difference between a linear and non linear differential equation. [2 marks]

ii) Show that the differential equation

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} = 0$$

and find the family of solutions to the differential equations. [6 marks]

Question three

a) Find the general solution of the system

$\dot{x}_1 = 5x_1 + 2x_2 + 2x_3$

$\dot{x}_2 = 2x_1 + 2x_2 - 4x_3$

$\dot{x}_3 = 2x_1 - 4x_2 + 2x_3$ [10 marks]

b) Determine the nature of the singular points of the differential equation below and solve it using an appropriate series method.

$xy'' + (1 - x)y' + 2y = 0$ [10 marks]

Question four

a) i) State the existence and uniqueness theorem for an nth order linear equation.

ii) Show that there exists a unique solution for the equation

$$\text{Cos } x\, y'' + x^2\, y' - xy = \sin x$$

where $x \neq \pi/2$ [6 marks]

b) Show that y = x is a solution of differential equation

$$x^2 y''' - 3x^2 y'' + x(6 - x^2) y' - (6 - x^2) y = 0$$

By reducing the order find the complete solution of the difference. [8 marks]

c) Show that the solutions to the differential equation

$$d^3y/dx^2 - 6d^2y/dx^2 = 5dy/dx + 12y = 0$$

are linearly independent on the interval -∞ < x < ∞ and write the general solution of this equation. [6 marks]

<u>Question five</u>

a) i) By considering the solution to the Legendre differential equation, define Legendre polynomials.

ii) State Rodrigues formula and use it to find the first three Legendre polynomials.

[7 marks]

b) Use the relation

$(n+1)P_{n+1}(x) - (2n+1)s\, P_n(x) + n\, P_{n-1}(x) = 0$ and the polynomials obtained above to compute P_4. [4 marks]

c) i) Show that $\left(x^p\, Jp(x)\right)' = x^p\, J_{p-1}(x)$

ii) Write down the solutions to the Bessel equation $x^2\, y'' + xy' + \left(x^2 - 9/4\right) = 0$ in the form J_p.

iii) Given that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\, \text{Sin } x$

find the solution to the equation in (ii) above in terms of sinx and cosx. [9 marks]

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