KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 360: MULTIVARIATE STATISTICAL METHODS 1

DATE: Tuesday 6th December, 2011 **TIME:** 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS

Answer question one and any other TWO questions.

1. (a) A random vector $\underset{\sim}{x}$ has covariance matrix

$$\Sigma = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Find the eigenvalues of Σ [3 marks]

(b) Consider the following data matrix with three variables

$$x = \begin{pmatrix} 9 & 12 & 3 \\ 2 & 8 & 4 \\ 6 & 6 & 0 \\ 5 & 4 & 2 \\ 8 & 10 & 1 \end{pmatrix}$$

Calculate (i) mean vector [2 marks]

(ii) variance matrix [3 marks]

(iii) correlation matrix [3 marks]

(c) Let $\underset{\sim}{x}' = (x_1, x_2, x_3)$ with mean $\mu_{\mu}^{1} = (2, 4, -1)$ and

$$\text{covariance } \Sigma = \begin{pmatrix} 4 & -1 & \tfrac{1}{2} \\ -1 & 3 & 1 \\ \tfrac{1}{2} & 1 & 6 \end{pmatrix}$$

Find the mean and variance of

(i) $Y_1 = x_1 + 2x_2 - x_3$

(ii) $Y_2 = 3x_1 - 4x_2$

Are Y_1 and Y_2 independent? Give reasons. [4 marks]

(d) Let $\underset{\sim}{x}$ have covariance matrix

$$\Sigma = \begin{pmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{pmatrix}$$

Determine the standard deviation matrix D and the correlation matrix p.

[6 marks]

(e) Let $\underset{\sim}{x} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ with $\mu_1^1 = (-3, 1, 4)$ and $\Sigma = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

(i) Obtain the distribution of $3x_1 - 2x_2 + x_3$ [3 marks]

Which of the following random variables are independent. Explain.

(ii) x_1 and x_3 [2 marks]

(iii) x_2 and x_3 [2 marks]

(iv) (x_1, x_2) and x_3 [2 marks]

Q2. (a) Let $\underset{\sim}{x}' = (x_1, x_2 ... x_p)$ be a p-dimensional normal density with mean $\underset{\sim}{\mu}$ and variance Σ.

Partition $\underset{\sim}{x}$ into subvectors $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ with orders q x 1 and (p – q) x 1 respectively.

Write down the conditional density function of $\underset{\sim}{x}_1$ given $\underset{\sim}{x}_{(2)}$ [3 marks]

(b) Let $\underset{\sim}{x} \sim N_3\left(\underset{\sim}{\mu}, \ \Sigma\right)$ where $\underset{\sim}{\mu}' = (1, 2, 1)$ and $\Sigma = \begin{pmatrix} 1 & 0.8 & -0.4 \\ 0.8 & 1 & -0.56 \\ -0.4 & -0.56 & 1 \end{pmatrix}$

Obtain the conditional expectation and the conditional variance of x_1 and x_3 given x_2.

[13 marks]

(c) If $\underset{\sim}{x}$ is distributed as $N_4\left(\underset{\sim}{\mu}', \Sigma\right)$ with $\underset{\sim}{\mu}' = (2, 1, 0, 1)$ and covariance matrix

$$\Sigma = \begin{pmatrix} 2 & -1 & 0 & 1 \\ -1 & 1 & 1 & -2 \\ 0 & 1 & 2 & 1 \\ 1 & -2 & 1 & 1 \end{pmatrix}$$

Find the distribution of $\underset{\sim}{Y}' = (x_1, x_3)$ [4 marks]

Q3. (a) Define the characteristic function of a random vector $\underset{\sim}{x}$ [2 marks]

(b) Let the random vector $\underset{\sim}{x}' = (x_1, x_2, ... x_p)$ be normally distributed with mean vector $\underset{\sim}{\mu}$ and covariance matrix $\Sigma > 0$.

(i) Find the characteristic function of

$$\underset{\sim}{z} = \Sigma^{-1/2}\left(\underset{\sim}{x} - \underset{\sim}{\mu}\right)$$ [5 marks]

Hence (ii) Obtain the characteristic function of $\underset{\sim}{x}$ [5 marks]

(iii) Defining $\underset{\sim}{Y} = \underset{\sim}{a}'\,\underset{\sim}{x}, \underset{\sim}{a} \neq 0,$ use the characteristic function technique to show that

$\underset{\sim}{Y} \sim Np\left(\underset{\sim}{a}'\,\underset{\sim}{\mu}, \underset{\sim}{a}\,\Sigma\,\underset{\sim}{a}\right)$ where $\underset{\sim}{a}$ is a vector of constants. [5 marks]

(c) Let $\underset{\sim}{x} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ with $\underset{\sim}{\mu}' = (2, -3, 1)$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$

Obtain the distribution of $2x_1 - 3x_2 + x_3$ [3 marks]

Q4. (a) Given the data

xj	10	5	7	19	11	8
y	15	9	3	25	7	13

Fit the linear regression model.

$Y_j = \beta_0 + \beta_1 x_{j1} + \ell_j,\ j = 1, 2, \ldots. 6$

Specifically, calculate the least squares estimates $\underset{\sim}{\hat{\beta}}$, the fitted values $\underset{\sim}{\hat{y}}$, the residuals $\underset{\sim}{\hat{\ell}}$ and the residual sum of squares $\underset{\sim}{\hat{\ell}}'\,\underset{\sim}{\hat{\ell}}$. [10 marks]

(b) Define the following theorems with respect to a random vector

(i) Central limit theorem

(ii) Weak Law of Large numbers. [4 marks]

(c) Given $\underset{\sim}{x} \sim N_3\left(\underset{\sim}{\mu}, \Sigma\right)$ with $\underset{\sim}{\mu} = 0$

and $\Sigma = \begin{pmatrix} 1 & 0.6 & -0.3 \\ 0.6 & 1.0 & -0.25 \\ -0.3 & -0.25 & 1.0 \end{pmatrix}$

Find the multiple correlation coefficient between x_1 and (x_2, x_3) [6 marks]

Q5. (a) Define (i) Partial correlation

(ii) Multiple correlation

(b) The multivariate normal random variable $\underline{X}$ is distributed with mean vector $\underset{\sim}{\mu} = \underset{\sim}{0}$ and

has covariance matrix $\Sigma = \begin{bmatrix} 6.64 & 0.80 & -0.44 & 0.38 \\ 0.80 & 10.86 & 14.09 & 11.32 \\ -0.44 & 14.09 & 2363 & 17.19 \\ 0.38 & 11.32 & 17.19 & 21.85 \end{bmatrix}$

(i) Let $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}$ be partitioned into

$\underline{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ and $X_2 = (X_4)$

Obtain the partial correlation between $\underline{X}_1$ given X_4.

(ii) Obtain multiple correlation between X_4 and the variates of $\underline{X}_1$

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