KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 361: THEORY OF ESTIMATION

DATE: WEDNESDAY, 30TH NOVEMBER 2011 **TIME: 2.00 P.M. – 4.00 P.M.**

Attempt question one and any other two questions.

Question one(30 marks)

a)Define the following terms: Statistic, Unbiasedness , Consistent estimator, confidence limits, prior probability (5 marks)

b)
(i)Let X be a random variable from the normal distribution with mean μ and variance σ^2. Show that

$$\bar{x} = \frac{\sum_i x_i}{n} \quad and \quad S^2 = \frac{\sum_i \left(x_i - \bar{x}\right)^2}{n-1}$$

are unbiased estimators of mean and variance respectively. (5 marks)

(ii)A random sample of five values are taken from a population with distribution $N(\mu,\sigma^2)$. The values are 1.06, 1.03,1.04, 1.05 1.04. Find unbiased estimators for μ and σ^2. (2 marks)

c)A coin is tossed 100 times. It lands heads up 64 times. Estimate p the probability of heads up and the variance of the estimate. (3 marks)

d)In an investigation to estimate the weight of 15 year old children, a random sample of 100 15-year old children is selected. Previous studies indicate that the variance of the weight of such children is 30 kg^2. Suppose that the sample mean weight is 38.4 kg, obtain the 95% confidence interval for μ. (5 marks)

e)Let $X_1, X_2, \ldots X_n$ be a random sample from a population with mean μ and variance σ^2 . Show that $\sum_{i=1}^{n} a_i x_i$ is an unbiased estimator of μ for any set of known constants $a_1, a_2, \ldots, a_n$, satisfying $\sum_{i=1}^{n} a_i = 1$

(5 marks)

f)Let

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu_0)^2}{2\sigma^2}}, & -\infty \le x \le \infty \\ 0, & otherwise \end{cases}$$

Where μ_0 is a known constant. Find the maximum likelihood estimator of σ^2. (5 marks)

Question two(20 marks)

a) Given that {3,2,5,1,4} is a random sample from the uniform distribution over the interval (λ-θ , λ+θ)
Find the moments estimates for λ and θ. (10 marks)

b) Given a random sample of size n from a population whose p.d.f is

$$f(x,\theta)=\begin{cases}\frac{1}{\theta}e^{-x/\theta}, & x>0\\ 0, & otherwise\end{cases}$$

Where θ is a positive constant. Obtain the maximum likelihood estimator of θ and test it for unbiasedness and consistency. (10 marks)

Question three(20 marks)

a) State the Cramer-Rao inequality. (3 marks)

b) Show that $E\left[\frac{\partial}{\partial\theta}\log f(x,\theta)\right]^2 = -E\frac{\partial^2}{\partial\theta^2}\log f(x,\theta)$ (6 marks)

c) Let X be r random variable such that

$$f(x)=\begin{cases}\theta e^{-\theta x}, & 0\le x\le\infty\\ 0, & otherwise\end{cases}$$

Find the Cramer-Rao lower bound for the variance of unbiased estimator of θ (11 marks)

d)

Question four(20 marks)

a) Let $x_1, x_2, \ldots x_n$ be a random sample of size n from a normal population with mean μ and variance σ^2, both unknown. Find the 100(1-σ)% confidence intervals for both μ and σ^2. (10 marks)

b) A random sample of size 15 from a normal population yields $\sum_{i=1}^{15} x_i = 48$ and $\sum_{i=1}^{15} x_i^2 = 271.2$. If both mean μ and variance σ^2 are unknown, determine the 95% confidence interval for σ^2 (10 marks)

Question Five(20 marks)

a) (i)State the Rao-Blackwell theorem . Explain briefly how it is used in improving estimators. (2,3 marks)

(ii)Let X be a random variable from a Bernoulli distribution. Take a random sample of size n. Let X_1 be an estimator of the probability of obtaining a 0. Using the Rao-Blackwell theorem, obtain an estimator with a smaller variance. (5 marks)

b) Let $y_i=\beta_0+\beta_1 x_i+e_i$

Be the simple linear model. Obtain the least square estimators of β_0 and $\beta_1$and check whether they are biased or not. (5,5 marks)

****