KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 361: THEORY OF ESTIMATION

DATE: Thursday 5th April 2012 **TIME:** 11.00a.m – 1.00p.m

INSTRUCTIONS: Answer question one and any other two questions

Q1 (a) Distinguish between the following terms:

(i) point estimation and interval estimation

(ii) a statistic and a parameter

(iii) Loss function and Risk function

(iv) Admissible estimator and Minimax estimator (8marks)

(b) Let X_1, X_2 be a random sample of size 2 from a normal distribution with mean θ and variance 1. Consider the following three estimators of θ:

$$t_1 = \tfrac{2}{3}X_1 + \tfrac{1}{3}X_2$$

$$t_2 = \tfrac{1}{4}X_1 + \tfrac{3}{4}X_2$$

$$t_3 = \tfrac{1}{2}X_1 + \tfrac{1}{2}X_2$$

(i) Find out if t_1, t_2 and t_3 are unbiased for θ.

(ii) Calculate the variances of the three estimators and comment on the results obtained.

(6marks)

(c) A random sample is taken from a normal population with unknown mean μ and variance 6.8. Suppose 8 sample values give the following statistics:

$\sum X_i = 4.22$ and $\sum X_i^2 = 2.6454$

Obtain the 98% confidence interval for μ (4marks)

(d) The weight of a new package of fat is known to have a mean of 40 grams. A sample of 7 packages yielded the following weights: 37, 38, 39, 40, 41, 38 and 42 grams.

If σ^2 is the standard deviation of the weights, calculate the 95% confidence interval of σ^2. Use α =0.05 (6marks)

(e) Three objects A, B and C with weights ω_1, ω_2 and ω_3 respectively are weighed on a spring balance. A and B together register 75g A and C register 45 g and B and C together 100g. Assuming the weights are independent with a constant variance, obtain the least square weights ω_1, ω_2 and ω_3. (6marks)

Q2 (a) Given the random sample of size n from the population

$f(x) = \dfrac{e^{-\theta}\theta^{x}}{x!}$, x = 0, 1, ...Obtain a sufficient statistic for θ that is also consistent

(7 marks)

(b) Let $X_1, X_2, ...X_n$ be a random sample of size n from a uniform distribution over the interval ($\mu - \sigma\sqrt{3}$, $\mu + \sigma\sqrt{3}$).

Find the moment estimators of μ and σ.

If a random sample of four values are taken from such a population, evaluate the moment estimators of μ and σ given that the four values are 50, 30, 11 and 41 (13 marks)

Q3 (a) consider the general linear model $Y_i = \alpha + \beta x_i$ such that $E(Y_i) = \alpha + \beta x_i$ and $Var(Y_i) = \sigma^2$ .Show the least square estimators, $\hat{\alpha}$ and $\hat{\beta}$ are unbiased for α and β respectively. (6 marks)

(b) A random sample $X_1, X_2, ...X_n$ of size n is taken from a population whose probability density function given by

$$f(x,\theta) = \begin{cases} \dfrac{1}{\theta} e^{-x/\theta} & ,x > 0 \\ 0 & ,elsewhere \end{cases}$$

where θ is a positive constant. Obtain the maximum likelihood estimator of θ and examine it for unbiasedness and consistency. (14marks)

Q4 (a) Let $X_1, X_2, ...X_n$ be a random sample of size n from a population whose p.d.f is f(x, θ) where θ is unknown. Let T=t($X_1, X_2, ...X_n$) be unbiased estimator θ. Under some regularity conditions to be stated, prove that the necessary and sufficient condition for the existence of Minimum Variance Unbiased estimator of θ is that the likelihood function can be expressed in the form $\frac{\partial LogL}{\partial \theta}=\frac{T-\theta}{\lambda}$.

Show also that Var(T) = λ (15marks)

(b) Let $X_1, X_2, ...X_n$ be a random sample of size n from a population

whose p.d.f is $f(x,\mu)=\begin{cases}\frac{e^{-\mu}\mu^x}{x!} & ,x=0,1,2,......\\ 0 & ,elsewhere\end{cases}$

Obtain the Cramer Rao Lower bound for $e^{-\mu}$ (5marks)

Q5 (a) Let $X_1, X_2, ...X_m$ and $Y_1, Y_2, ...Y_n$ be two independent random samples from normal populations X~N(μ_x, σ^2) and Y~N(μ_y, σ^2) respectively. Derive the 100(1-α)% confidence intervals for the difference of the means $\mu_x - \mu_y$.

(11marks)

(b) Two analysts took repeated readings on the hardness of city water. Assuming normal populations with common variance, Calculate a 95% confidence interval for $\mu_x - \mu_y$

Analyst I(X)	0.42	0.04	0.27	0.39	0.34	0.68		
Analyst II(Y)	0.72	0.51	0.99	0.61	0.23	0.48	0.33	0.35

(9marks)