KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SMA 400: TOPOLOGY I

DATE: Thursday, 29th March, 2012 TIME: 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS: Answer question **ONE** and any other **TWO** questions.

QUESTION ONE (30 MARKS)

a) i) Let X be a non-empty set. Define what is meant by a topology on X (3marks)

ii) Let □ be the set of all natural numbers and let τ_4 consist of □ , $\varnothing$ and all finite subsets of □ . Determine whether τ_4 is a topology on □ . (3 marks)

b) Let (X,τ) be a topology space and $X=\{a,b,c,d,e,f\}$, $\tau=\{\varnothing, X, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d,e,f\}\}$. Find

i) Sets that are open but not closed (2 marks)

ii) Sets that are neither open nor closed (2 marks)

ii) Sets that are closed but not open (2 marks)

c) Show that the intersection of any (finite or infinite) number of closed sets in a topological space (X,τ) is a closed set (5 marks)

d) i) Define what is meant by limit point of A, a subset of a topological space (X,τ) (2 marks)

ii) Determine A' given that $X=\{a,b,c,d\}$ and $\tau=\{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ is a topology on X (3 marks)

e) Consider the topology $\tau=\{X,\varnothing,\{a\},\{c,d\},\{a,c,d\},\{b,c,d,e\}\}$ on $X=\{a,b,c,d,e\}$ and the subset $A=\{a,d,e\}$ of X. Determine,

i) A^0

ii) $ext(A)$

iii) $b(A)$ (6 marks)

f) Determine the neighbourhood system of a point p in an indiscrete space (2 marks)

QUESTION TWO (20 MARKS)

ii) Consider $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x)=\begin{cases} x-1, & if\ x\le 3 \\ \frac{1}{2}(x+5), & if\ x>3 \end{cases}$$

Determine whether or not f is continuous (3 marks)

b) Prove that $f(A\cup B)=f(A)\cup f(B)$ (5 marks)

c) Show that the function $d:\mathbb{R}\times\mathbb{R}\to\mathbb{R}$ given by $d(a,b)=|a-b|,\ a,b\in\mathbb{R}$ is a metric on the set $\mathbb{R}$ (4 marks)

d) Let (X,τ) be a topological space with $X=\{1,2,3\},\ \tau=\{\varnothing,X,\{1\},\{2\},\{1,2\}\}$. Show that (X,τ) is a normal space. (6 marks)

QUESTION THREE (20 MARKS)

a) Let A be a subset of a topological space (X,τ). Show that A is closed in (X,τ) if and only if A contains all of its limit points (6 marks)

b) i) Define neighbourhood of a point p in a topological space (X,τ) (2 marks)

ii) Determine the neighbourhood system of a point p in an indiscrete space (3 marks)

c) Let A and B be subsets of X, a topological space. Show that $(A\cup B)'=A'\cup B'$ (5 marks)

d) Prove that if (X,τ) is a topological space such that, for every $x\in X$, the singleton set $\{x\}\in\tau$, then τ is the discrete topology. (4 marks)

QUESTION FOUR (20 MARKS)

a) Prove that the intersection of any two topologies τ_1 and τ_2 is also a topology on X. (7marks)

b) Consider the subset $A=\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\ldots\right\}$ of $\mathbb{R}$ with the topology of open sets (a,b).

Determine whether or not A is nowhere dense in $\mathbb{R}$. (3marks)

c) i) Define what is meant by a basis for a topology τ. (2marks)

ii) Let β be the collection of all all open-half intervals of the form $(a,b]$, $a\angle b$. Show that β is a basis for a topology on □ . (3marks)

d) Show that $A \cup A'$ is a closed set, where A is a subset of a topological space (X,τ) (5marks)