KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 401: TOPOLOGY 2

DATE: MONDAY 2ND APRIL 2012 TIME: 2.00 P.M. – 4.00 P.M.

INSTRUCTIONS: Attempt Question ONE and any other Two.

Question 1 (30 Marks)

a.

i) Define a cover for a set A. [2 marks]

ii) Hence or otherwise state the Heine- Borel theorem. [2 marks]

b. By use of an example show that a subset of a compact space need not be compact. [3 marks]

c. Let X and Y be topological spaces and $f : X \to Y$ be continuous. Let A be a compact set in X. Prove that $f(A)$ is compact in Y. [8 marks]

d.

i) When is a class $\{A_i\}$ of subsets of a topological space X said to have the finite intersection property? [3 marks]

ii) Hence give an example of a class that has the finite intersection property. [2 marks]

e. Define compactness in terms of closed sets. [2 marks]

f.

i) Let X and Y be topological spaces. Define a product topology on the cartesian product $X \times Y$ [4 marks]

ii) Let $X = \{a,b,c\}$ and $Y = \{d,e\}$. Find $X \times Y$ [2 marks]

g. Give the definition of a connected topological space. [2 marks]

Question 2 (20 Marks)

a.

i) What conditions must be satisfied by a set say A for the Heine Borel theorem to be satisfied? [2 marks]

ii) Hence by use of examples show that each of those two conditions must be satisfied by a set A or else the Heine –Borel theorem will not be satisfied. [6 marks]

b.

i) Define an open cover for a set A. [2 marks]

i) Hence define a compact subset A of a topological space X. [2 marks]

ii) Show that the open interval $A = (0,1)$ on $\mathbb{R}$ with the usual topology is not compact. [8 marks]

Question 3 (20 Marks)

a. Let (X,τ) be a topological space.

i) Define the term sequentially compact subset of (X,τ). [2 marks]

ii) Hence show that the open interval $A=(0,1)$ on $\mathbb{R}$ with the usual topology is NO T sequentially compact. [5 marks]

b.

i) Let X be a topological space. Define a countably compact set in X. [2 marks]

ii) Show by use of an example that a countably compact set need not be compact.. [5 marks]

c.

i) State what is meant by a topological property in a topological space. [2 marks]

ii) Hence show that connectedness is a topological property. [4 marks]

Question 4 (20 Marks)

a.

i) Define cofinite topology. [2 marks]

ii) Prove that the set of integers is connected in the cofinite topology. [5 marks]

b. Consider the following subset of the plane $\mathbb{R}^2$

X = $\{x \times y : y = 0\} \cup \left\{x \times y : x > 0 \text{ and } y = \frac{1}{x}\right\}$. Is X connected or not? [1 mark]

Explain. [2 marks]

c. Prove that if the sets A and B form a separation of X and if Y is a connected subspace of X then Y lies entirely within either A or B. [5marks]

d.

i) Define a simply connected topological space. [2 marks]

ii) Hence give an example of a space that is simply connected and another that is not simply connected.. [3 marks]

Question 5 (20 Marks)

a. Show that if A and B are non-empty separated sets, then $A \cup B$ is disconnected [5 marks]

b. State without proof the Tychonoff's product theorem. [2 marks]

c. The axiom of choice states that given non empty sets A_i for $i \in I$, the product $\prod_{i\in I} A_i \neq \varnothing$. Proof this statement using Tychonoff's product theorem. [5 marks]

d. Consider the topology $\tau = \{X, \varnothing, \{c\}, \{a.b\}\}$ on $X = \{a,b,c\}$ and the topology $\tau^{*} = \{Y, \varnothing, \{u\}\}$ on $Y = \{u,v\}$.

i) Determine the defining subbase β_s of the product topology on $X \times Y$.

[5 marks]

ii) Determine the defining base β_s for the product topology on $X \times Y$.

[3 marks]