KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 404: COMPLEX ANALYSIS II

DATE: Wednesday 30th November, 2011 **TIME:** 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS: ***Answer question one and any other TWO questions.***

QUESTION ONE (30 MARKS)

a) i. Define an isolated singular point $z = z_0$ (1 marks)

ii Locate all the singular points of the function $f(z) = \dfrac{z+2}{z^2(z^2+9)}$ (1 marks)

iii. Determine whether the singular points in (ii) above are isolated or not. Justify your answer (3 marks)

b) i. Obtain the Laurent series of the function $f(z) = (z-5)\sin\left(\dfrac{1}{z-2}\right)$ (3 marks)

ii Use the Laurent series obtained in (i) above to find the residue of $f(z) = (z-5)\sin\left(\dfrac{1}{z-2}\right)$ at $z = 2$ (1 marks)

iii State the type of the singular point and justify your answer. (2 marks)

c) Evaluate the following integrals

i. $\displaystyle\int_0^{2\pi} \frac{\sin^2\theta}{5+4\cos\theta}\, d\theta$ (5 marks)

ii. $\displaystyle\int_0^{\infty} \frac{\sin 2x}{(x^2+1)^2 x}\, dx$ (5 marks)

d) Prove that $f_2(z) = \dfrac{1}{1+i}\displaystyle\sum_{n=0}^{\infty}\left(\frac{z+i}{1+i}\right)^n$ is an analytic continuation of $f_1(z) = \displaystyle\sum_{n=0}^{\infty} z^n$, showing graphically the regions of convergence of the series. (5 marks)

e) Find the residue of $f(z) = \dfrac{\cot z \coth z}{z^3}$ at $z = 0$ (4 marks)

QUESTION TWO (20 MARKS)

a) Evaluate the contour integral $\displaystyle\int_C \left(z e^{3/z} + \frac{\cos z}{z^2(z-\pi)^3} \right) dz$ where $C : |z| = 5$ (6 marks)

b) Use residues to find the partial fraction representation of $f(z) = \dfrac{z^2 - 7z + 4}{(z+4)^3(z-2)^2}$ (6 marks)

c) Evaluate the following integral involving multiple-valued function $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{x}\,(x+4)}\, dx$ (5 marks)

d) Evaluate the integral $\int_c \frac{z+1}{z^2-2z}dz$,$|z|=3$, by getting a single residue of related function $1/z^2 f(1/z)$ at $z=0$. (3 marks)

QUESTION THREE (20 MARKS)

a) Show that the mapping $f(z)=\cos z$ is conformal at the point i.Find the angle of rotation and scale factor at the point. (5 marks)

b) Consider triangle A, B, C bounded by the curves $c_1: x=1, c_2: y=1$ and $c_3: x+y=1$.Determine the region of the w-plane into which the above triangle is mapped by $f(z)=z^2$.Is $f(z)$ conformal at (1, 1)? (10 marks)

c) Show that if $f(z)$ has a pole of order m at z_0,then residue at z_0 is given by

$$\frac{1}{(m-1)!}\lim_{z \to z_0}\frac{d^{m-1}}{dz^{m-1}}(z-z_0)^m f(z)$$ (5 marks)

a) i. State the Schwarz-Christoffel transformation (4 marks)

ii. Show that when $x_n=\infty$,the last factor in the Schwarz-Christoffel transformation is 1. (4 marks)

b) Which of the following functions satisfy the Schwarz reflection principle

i. $f(z)=z^2+1$ (4 marks)

ii. $f(z)=e^{iz}$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate the following integrals

i. $\int_{-\infty}^{\infty}\frac{x^2}{(x^2+1)^2}dx$ (4 marks)

ii. $\int_0^{\infty}\frac{\ln x}{x^2+4}dx$ (6marks)

b) Use Scwarz-Christoffel transformation to find a function which maps the upper half plane on to the interior of a triangle with vertices at $w=0,1,i$ corresponding to $z=0,1,\infty$. (6 marks)

c) Use the expression $v(x,y)=\int_{(x_0,y_0)}^{(x,y)}-u_t(s,t)ds+-u_s(s,t)dt$, to find a harmonic function $v(x,y)=x^3-3xy^2$.Write the resulting analytic function in terms of the complex variable z. (4 marks)