KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 404: COMPLEX ANALYSIS II

DATE: WEDNESDAY, 4TH APRIL 2012 **TIME: 11.00 A.M. – 1.00 P.M.**

INSTRUCTIONS: Answer question ONE and any other TWO questions

Question One (30 marks)

a) Define the following terms:

(i) Singular point

(ii) Isolated Singular point

(iii) Principal part of a function

(iv) Simple pole

(4 marks)

b) Locate all the singular points of the function:

$$f(z)=\frac{z+2}{z^2\left(z^2+9\right)}$$ (4 marks)

c) Obtain the Laurent series expansion for

$$f(z)=\frac{e^{-z}}{(z+1)^2}$$ (3 marks)

d) Evaluate the following integrals

$$\int\frac{xdx}{x^3-8}$$ (5 marks)

e) Obtain the residues of the following function

$$f(z)=\frac{2z^2-3z-1}{(z-1)^3}$$ (4 marks)

(f) Let rectangular region R in the z plane be bounded by $x = 0, y = 0$, $x = 1, y = 1$.
Determine the region R of the W plane into which R is mapped under the transformations
$W = z + (1 - 2i)$. (4 marks)

(g) Prove that $f_1(z) = z - z^2 + z^3 - z^4$ is analytic in $|z| < 1$ and find a function that represents all possible analytic continuation of $f_1(2)$. (6 marks)

Question Two (20 marks)

a) Obtain the integral of the following function.

$$\int_c \frac{5z-2}{z(z-1)} = dz$$

(6 marks)

b) Find the residues of the following function

$f(z) - \tan z \qquad |z| = 2.$

Use it to evaluate

$$\int_c \tan z \, dz.$$

(6 marks)

c) Use residues to find the partial fraction representation of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

(6 marks)

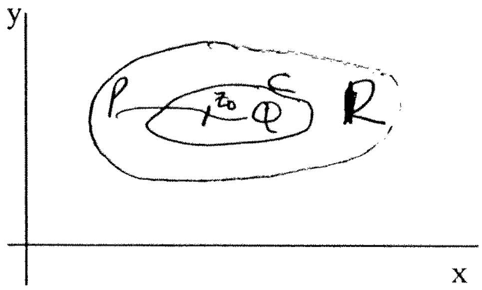
d) Differentiate between a residue and a pole. (2 marks)

Question Three (20 marks)

a) Prove that $f_1(z) = \int_0^\infty t^3 e^{-zt} dt$ is analytic at all points z in which Re $z>0$. Find an analytic continuation of $f(z)$ into the left plane Re $z<0$. (8 marks)

b) Let *F(2)* be analytic in a region R and suppose that *F(z)* = *0* at all points on an arc PQ inside R.

Prove that *F(z)* = *0* throughout R



(6 marks)

c) Given that the identity $\sin^2 z + \cos^2 z = 1$ holds for all real values of z, prove that it also holds for all complex values of z. (6 marks)

Question Four (20 marks)

a) Show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)} = \frac{\Pi}{6}$ (6 marks)

b) Evaluate $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2+4}\, dx$ (6 marks)

c) Integrate the following function

$$\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5 - 4\cos\theta}$$ (8 marks)

Question Five (20 marks)

a) If $C_1 : X = 1,\ C_2 ; y = 1$. Investigate conformality of $T(z) = z^2 + 2z + 1$ at $Z_0 = 1 + i$. (8 marks)

b) Determine the image of triangle ABC by *f(z)*= z^2. Is *f(z)* conformal at the vertices? (8 marks)

c) State the Schwartz-Christoffel transformation. (2 marks)

d) Define conformal transformation. (2 marks)

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