KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ARTS, BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

SMA 430: NUMERICAL ANALYSIS II

DATE: Monday, 2nd April, 2012 TIME: 8.00 a.m. – 10.00 a.m.

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions.

Question One

a) Solve the linear system of equations below by determining the inverse of the coefficient matrix

$$x+2y-z=2$$
$$3x+6y+z=1$$
$$3x+3y+2z=3$$

(6 marks)

b) Approximate $f'(3.15)$ using $h=\pm 0.1$ using the three point formula with the following data

x	2.95	3.05	3.15	3.25	3.35
f(x)	13.364875	14.665125	16.088375	17.6400625	19.327875

(4 marks)

c) Solve the following system using Grauss elimination method and pivoting.

$$\begin{aligned} x_1+x_2+x_4 &= 2\\ 2x_1+x_2-x_3+x_4 &= 1\\ -x_1+2x_2+3x_3-x_4 &= 4\\ 3x_1-x_2-x_3+2x_4 &= -3 \end{aligned}$$

(6 marks)

d) Use Euler's method to approximate the solution to the initial value problem

$dy/dt = \frac{1}{t}\left(y^2+y\right), 1 \le t \le 3,\ y(1.0) = -2\ with\ h = 0.5$ (5 marks)

e) Evaluate the integral $F = \int_0^1 \frac{1}{1+x}dx$ by sub-dividing the interval $[0,1]$ into two parts i.e. $[0,0.5]\ to\ [0.5,1.0]$ and then apply the Grauss – legendre three point formula $(n=2)$. Compare your answer with the solution from direct integration.

(5 marks)

f) Find the first two iterations of the Jacobi method for he linear system below using $\underset{\sim}{x}^{(0)} = (0,0,0)^t$.

$$\begin{aligned} 10x_1-x_2 &= 9\\ -x_1+10x_2-2x_3 &= 7\\ -2x_2+10x_3 &= 6 \end{aligned}$$

(4 marks)

Question Two

a) Make an LU decomposition of the matrix

$$A = \begin{pmatrix} 3 & 6 & 9\\ 1 & 0 & 5\\ 2 & -2 & 16 \end{pmatrix}$$

and hence solve $Ax = b\ where\ b = (21,9,28)^T$ (10 marks)

b) i) Compute the condition number (A) of the matrix

$$A\begin{pmatrix} 3.9 & 1.6\\ 6.8 & 2.9 \end{pmatrix}$$

ii) The linear system

$$\begin{aligned} 3.9x_1+1.6x_2 &= 5.5\\ 6.8x_1+2.9x_2 &= 9.7 \end{aligned}$$

has the actual solution $x = (1,1)^t$ and an approximate solution

$\tilde{x} = (0.98, 1.1)^t$

Compute the error estimates $\left\| x - \tilde{x} \right\|_\infty$ and $K_\infty(A) \dfrac{\left\| b - A\tilde{x} \right\|_\infty}{\left\| A \right\|_\infty}$

(10 marks)

Comment on the sensitivity of the above system to round off errors.

Question Three

a) Given the following values of a function $f(x)$ where x is the variable, obtain a least square fit of the form $f(x) = ae^{-3x} + be^{-2x}$.

xi	0.1	0.2	0.3	0.4
fi	0.76	0.58	0.44	0.35

(10 marks)

b) Obtain the Chebysher Polynomial approximation of degree one to the function $f(x) = x^3$ on $[0,1]$. (6 marks)

c) Evaluate the integral $\int_{-1}^{1} (1 - x^2)^{\frac{3}{2}} \cos x d$. Using the Gauss – Legendre three point $(n=2)$ integration formula (4 marks)

Question Four

a) Consider the initial – value problem $dy/dx = y + x,\ y(1.0) = 1.0$. Use the classical fourth order Runge – Kutta method with $h = 0.1$ to solve for $y(1.2)$

(10 marks)

b) i) Use Taylor's method of order two to approximate the solution of the initial – value problem $dy/dt = \sin t + e^{-t},\ 0 \le t \le 1,$

$$y(0) = 0 \text{ with } h = 0.5$$

ii) Use direct integration to evaluate ${}^{1}\int_{0}\left(\sin t + e^{-t}\right)dt$ and compare your answer with the one in (i) above.

(10 marks)

Question Five

a) Prove that the iteration method of the form $x^{(k)} = Tx^{(k-1)} + C$ for each $k \geq 1$ *and* $C \neq 0$ for the solution $Ax = b$ converges to the exact solution for any initial vector $x \in \Re^{n}$ if $P(T) < 1.$ (4 marks)

b) Show that the iterative matrix for the SOR method is

$$x^{(k+1)} = (D - WL)^{-1}((1-W)D + WU)x^{k} + W(D - WL)^{1}_{b}$$ (4 marks)

c) For the solution of the system of equations

$$\begin{aligned} 2x - x_2 &= 7 \\ -x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 2x_3 &= 1 \end{aligned}$$

Set yup the Gauss – Seidel and SOR iterative schemes for the solution and iterate three times starting with $x^{(0)} = 0,$ *use* $W = 1.25.$ Compare with the exact solution And find also the optimum relaxation factor for the SOR. (12 marks)