KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

INSTITUTIONAL BASED PROGRAMME (IBP) (DECEMBER SESSION)

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 431: DIFFERENTIAL GEOMETRY

DATE: **Wednesday 2nd May 2012** **TIME:** **8.00a.m – 10.00a.m**

INSTRUCTIONS:

Answer question ONE and any TWO of the remaining questions. Show all the necessary working

QUESTION ONE (30 MARKS)

a) Define direction *cosine* of a vector $\bar{a}$ show that

$$Cos\theta_l = \frac{a_l}{|\bar{a}|}. \quad i = 1,2,3$$ [4marks]

b) The graph of the equation
$r = 2\,Cos\,\theta - 1$, $0 \leq \theta \leq 2\pi$ in polar coordinates represents a curve. Show that this representation is regular. [4marks]

c) Find the equation of the tangent line and normal plane to the curve
$x_1 = (1 + t)$, $x_2 = -\,t^2$, $x_3 = (1 + t^3)$ [5marks]

d) Define curvature vector of a curve.

Find the curvature vector $\bar{k}$ and curvature k on the curve

$$\bar{x} = t\,\bar{e}_1 + \tfrac{1}{2}t^2\,\bar{e}_2 + \tfrac{1}{3}t^3\,\bar{e}_3$$ [7marks]

e) Show that a curve of class ≥ 3 along which $\bar{n}$ is of class C^1 *is* a plane curve if and only if its torsion vanishes identically. [5marks]

f) Find the equation of the tangent plane and normal line to the surface represented by

$$\bar{x} = u\,\bar{e}_1 + v\,\bar{e}_2 + (u^2 - v^2)\,\bar{e}_3$$

at the point corresponding to $u = 1, v = 1$. [5marks]

QUESTION TWO (20 MARKS)

a) i) Prove that if $\bar{x} = \bar{x}\ (s)$ is natural representation of a curve *C*, then $|s_2 - s_1|$ is the length of the arc segment of *C* between $\bar{x}\ (s_1)$ and $\bar{x}\ (s_2)$.

ii) If $\bar{x} = \bar{x}\ (s^*)$ is any other natural representation of *C*, then prove that.

$s = \pm\, s^* +$ Constant [8marks]

b) Introduce arc length *s* as a parameter along
$\bar{x} = (e^t \cos t)\ \bar{e}_1 + (e^t\ \sin t)\bar{e}_2\ \ + e^t\ \bar{e}_3\ \ ,\ \ \infty < t < \infty$ [4marks]

c) Find the tangent vector along the helix
$\bar{x} = a\ \cos t\ \ \bar{e}_1 + a\ \sin\ \bar{e}_2\ \ + bt\ \bar{e}_3\ ,$
where *a* and *b* are non-zero. [4marks]

d) Find the intersection of the $x_1\ x_2$ plane and the tangent line to the helix
$\bar{x} = \ \cos t\ \bar{e}_1 + (\sin t)\bar{e}_2\ \ + t\,\bar{e}_3\ (t > o)$ [4marks]

QUESTION THREE (20 MARKS)

a) Show that the curvature is independent of orientation. [4marks]

b) Prove that a regular curve of class ≥ 2 is a straight line if and only if its curvature is identically zero. [6marks)

c) Find the equations of the principal normal and osculating plane at the point $t = \frac{\pi}{2}$ on the helix
$\bar{x} = \ \cos t\ \bar{e}_1 + \ \sin t\,\bar{e}_2\ \ +\ \bar{e}_3$ [10marks]

QUESTION FOUR (20 MARKS)

a) Along the curve $\bar{x} = \bar{x}\ (s)$, show that
$\dddot{\bar{x}} = k^2\bar{t}\ + \acute{k}\,\bar{n}\ \ +\ \ \tau\, k\, \bar{b}$ and hence
$[\ \dot{\bar{x}}\ ,\ \ddot{\bar{x}}\ ,\ \dddot{\bar{x}}\ ] = \ k^2\ \tau$, where k and τ are curvature and torsion respectively [10marks]

b) Show that the sign of torsion τ is independent of the sense of normal $\bar{n}$ and the orientation of C. [5marks]

c) Determine the torsion along the curve

$\bar{x} = (3t - t^3)\ \ \bar{e}_1 + 3t^2\,\bar{e}_2\ \ + (3t + t^3)\ \bar{e}_3$ [5marks]

QUESTION FIVE (20 MARKS)

a) Define a surface and write its parametric equation. [2marks]

b) Show that the first fundamental form on the surface

$\bar{x} = f(t)\cos\theta\ \bar{l} + f(t)\sin\theta\ \bar{j} + g(t)\ \bar{k}$

is $I = (f(t))^2 d\,\theta^2 + [(f'(t))^2 + (g'(t)^2]\ dt^2$ [10marks]

c) Find the second fundamental form on the surface

$\bar{x} = u\ \bar{e}_1 + v\ \overline{e_2} + (u^2 - v^2)\ \overline{e_3}$ [8marks]