KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2010/2011

INSTITUTE OF OPEN LEARNING

EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: FRIDAY, 4TH FEBRUARY 2011 **TIME: 8.00 A.M. - 10.00 A.M.**

INSTRUCTIONS: Answer Question One and any other two questions.

Question One (30 marks)

a) Reduce the equation

$3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ to its canonical form. (5 marks)

b) Define a boundary value problem and give two examples. (4 marks)

c) Consider the partial differential equation

$$x\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 3y^2\frac{\partial u}{\partial x} = 0$$

Determine the regions for which the equation is hyperbolic, parabolic and elliptic.

(5 marks)

d) Solve the two dimensional Laplace equation

$$u_{xx} + u_{yy} = 0$$

with the conditions

$$u_x(0, y) = 0,\ \ u_x(a, y) = 0$$
$$u(x,0) = x,\ \ u(x,b) = 0$$

(10 marks)

e) Verify that $u = f(x+y) + g(2x+y)$ is a solution of the partial differential equation $u_{xx} - 3u_{xy} + 2u_{yy} = 0$. (6 marks)

Question Two (20 marks)

a) Obtain the solution of the boundary value problem

$$u_{xy} = 4xy + e^x, \; u_y(0, y) = y, \; u(x,0) = 2$$ (4 marks)

b) Use the Laplace transform to solve the equation.

$$\frac{\partial^2 u}{\partial x \partial t} + \sin 2t = 0, t > 0$$

given that $u(x,0) = x, \; u(0,t) = 0$ (10 marks)

c) Consider the partial differential equation

$$u_{xx} + (x+2)u_{xy} + 2xu_{yy} = 0.$$

Reduce this equation to its canonical form. (6 marks)

Question Three (20 marks)

a) Using Fourier transform, find the solution of the two-dimensional Laplace equation

$$u_{xx} + u_{yy} = 0$$

in the region $y > 0, -\infty < x < \infty$ and so that $u(x,0) = f(x)$ (12 marks)

b) Verify that $u(x,t) = \theta(1 - erf\left(\frac{x}{2\sqrt{kt}}\right)\Big]$

solves the heat equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$ subject to the conditions $u(x,0) = 0, \; u(0,t) = \theta_0$, θ_0 being a constant. (8 marks)

Question Four (20 marks)

a) Reduce the equation

$2xy\frac{\partial^2 u}{\partial x^2} - 3x^2\frac{\partial^2 u}{\partial x \partial y} - 2y\frac{\partial u}{\partial x} = 0$ to its canonical form and hence find the solution

that satisfies $u = x^2, \frac{\partial u}{\partial x} = \frac{1}{3}x$ on $y = x.$ (10 marks)

b) Obtain the cylindrical polar form of the Laplace's equation in two dimensions.

(10 marks)

Question Five (20 marks)

Find the Riemann-Green function of the equation.

$$\frac{\partial^2 u}{\partial x \partial y} + \frac{1}{x+y}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right) = 0$$

Hence show that the solution of this equation which satisfies the conditions

$u = 0, \frac{\partial u}{\partial x} = f(x)$ on $y = x,$ is given by $u(x,y) = \frac{2}{x+y}\int_y^x \tau \, f(\tau)d\tau.$ (20 marks)

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