KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: Monday, 26th March, 2012 TIME: 11.00 a.m. – 1.00 p.m.-

INSTRUCTIONS:

Answer question **ONE** and any other **TWO** questions.

Question One (30 marks)

a) Verify that the function $u(x,t) = f\left(\frac{x+ct}{2}\right) + g\left(\frac{x-ct}{2}\right) - f(0)$

solve the initial value problem $u_{tt} = c^2 u_{xx}$

$$u(x,t) = g(x) \; on \; x+ct = 0$$

$$u(x,t) = f(x) \; on \; x-ct = 0$$

Where $f(0) = g(0)$. (6 marks)

b) Convert the equation

$2u_{xx} + 3u_{xy} + u_{yy} = 0$

to its canonical form (8 marks)

c) Find the d'Alembert's solution of the initial value problem $u_{tt} = c^2 u_{xx}$

subject to the conditions

$u(x,0) = \sin x$

$u_t(x,0) = \cos x$ (3 marks)

d) Solve the equation $\frac{\partial^2 u}{\partial x \partial y} = 4e^y \cos 2x$, given that at

$y=0, \frac{\partial u}{\partial x} \cos x \; and \; at \; x = \pi, u = y^2.$ (5 marks)

e) Find the Riemann – Green function for the equation $\frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - 2u = 0$

and hence obtain the solution that satisfies the conditions

$u = 0, \frac{\partial u}{\partial x} = 2x \; on \; y = x.$ (8 marks)

Question Two (20 marks)

a) Use Fourier Transform method to solve the equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}; t > 0$

given that $u(x,0) = f(x) \; -\infty < x < \infty$. (10 marks)

b) Using the Laplace transform method, solve the initial boundary value problem

described by

$$u_{tt} = u_{xx} \;\; 0 < x < l, \; t > 0$$
$$u(0,t) = u(l,t) = 0, \; t > 0$$
$$u(x,0) = \sin \pi x, \; u_t(x,0) = -\sin \pi x, \; 0 < x < l.$$

(10 marks)

Question Three (20 marks)

a) An infinitely long, string having one end at $x = 0$ is initially at rest on the x-axis. At $t = 0$ *the end* $x = 0$ begins to move along the u axis in the manner described by $u(0,t) = a \cos \sigma t$. Find the displacement $u(x,t)$ of the string at any point at any time. (10 marks)

b) A rod of length *l* with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0^oC and are kept at that temperature. Find the temperature at any point at any time along the rod. (10 marks

Question Four (20 marks)

a) Determine the region in which the equation $u_{xx} + x^2 u_{yy} = 0$ is elliptic and transform the equation in the region to canonical form. (8 marks)

b) Show that the equation $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = 0$ is parabolic everywhere. By reducing it to the canonical from, find its general solution. (12 marks)

Question Five (20 marks)

a) Verify that the function $u(x,y) = f\left(\sqrt{\dfrac{y^2+x^2-8}{2}}\right) + g\left(\sqrt{\dfrac{x^2-y^2+16}{2}}\right) - f(2)$

solves the characteristic initial value problem

$$xy^3 u_{xx} - x^3 y\, u_{yy} - y^3 u_x + x^3 u_y = 0$$
$$u(x,y) = y(x) \; on \; y^2 - x^2 = 8 \;\; for \;\; 0 \le x \le 2$$
(8 marks)
$$u(x,y) = f(x) \; on \; y^2 + x^2 = 16 \;\; for \;\; 2 \le x \le 4 \; with \; f(2) = g(2)$$

b) An infinitely long uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π; this end is maintained at a temperature u_0 at all points and other edges are at zero temperature. Write down the boundary value problem modeling the temperature distribution in steady state and hence find the temperature distribution. (12 marks)