KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2011/2012

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 463: TIME SERIES ANALYSIS

DATE: Tuesday 6th December, 2011 **TIME:** 11.00 a.m. – 1.00 p.m.

INSTRUCTIONS

Answer Question 1 and any other TWO questions.

Q1. (a) Define the following terms as used in time series analysis. [4 marks]

(i) Trend

(ii) Seasonal variations

(iii) Cyclic variations

(iv) Random effects. [4 marks]

(b) Explain briefly the four objectives of time series analysis. [4 marks]

(c) Why is a time plot regarded as an important first step in the analysis of a time series? [2 marks]

(d) Sketch a logistic curve and use it to explain the trend displayed by the output of many industries. [3 marks]

(e) Consider the time series

$X_t = \sum_{j=0}^{\infty} e_{t-j}$, where $\{e_t\}$ is a sequence of independent random variables with mean, p, and variance σ^2. Show that $Y_t = X_t - X_{t-2}$ is a stationary process. [5 marks]

(f) The following data shows profit (thousand Ksh) for four years on quarterly basis

Year	1st Q	2nd Q	3rd Q	4th Q
2007	75	60	54	59
2008	86	65	63	78
2009	91	73	67	85
2010	104	81	75	96

Find the trend value for the 3rd Quarter of 2009 using an appropriate moving average filter. [3 marks]

(g) Calculate the seasonal indices for the data given in 1(f) above assuming a multiplicative model. What does the seasonal index for the third quarter tell you about the profits? [5 marks]

(h) The following data is known to contain a second order polynomial trend. Obtain a series which is free of trend using an appropriate filter

t	0	1	2	3	4	5	6	7	8	9	10
x_t	22	32	81	41	92	67	45	57	24	83	28

[3 marks]

(i) Distinguish between a deterministic and a stochastic time series. [2 marks]

Q2. (a) A quadratic polynomial in t is fitted to 7 consecutive points of a time series. Estimate the trend value corresponding to the middle point of the set as a weighted moving average of the observed values. Hence find the trend value for 1964 given the following data. [10 marks]

Year	1960	1961	1962	1963	1964	1965	1966	1967	1968
Sales	1000	2000	4000	3000	6000	5000	4000	2000	6000

(b) Let $x_t = e^{i\lambda t}$ be the original time series.

Define $Yt = \Delta x_t = x_t - x_{t-1}$ where Δ is the difference operator. Show that

$Y_t = 2\mathrm{Sin}\, \lambda/_2\, e^{i(\lambda t + \phi(\lambda))}$ where $\phi(\lambda)$ is the phase angle show that this filter removes trend.

[10 marks]

Q3. (a) Consider an AR(2) given by

$x_t = \frac{1}{12} x_{t-1} + \frac{1}{12} x_{t-2} + \ell_t$ where $\{e_t\}$ is a purely random process. Is this process stationary. If so obtain its autocorrelation function. [10 marks]

(b) Show that an AR(1) process x_t, can be expressed as an infinite moving average process of the $\{e_t\}$ are uncorrelated (o, σ^2) random variables.

Hence find the autocovariance and the autocorrelation functions of x_t. [10 marks]

Q4. (a) Consider the process $x_t = \sum_{j=0}^{2} \ell_{t-j}$ where $\{e_t\}$ is a purely random process.

(i) Show that x_t is weakly stationary. [5 marks]

(ii) Find the spectral density of x_t. [5 marks]

(iii) Use (ii) above to find the spectral density of $Y_t = \sum_{j=-1}^{1} x_{t-j}$ [5 marks]

(b) Define $\{x_t: t = 0, \pm 1, \pm 2, \ldots\}$ by

$X_t = \ell_1 \operatorname{Sin}\lambda t + \ell_2 \operatorname{Cos}\lambda t$ where ℓ_1 and ℓ_2 are independent $(0, \sigma^2)$ random variables.

Show that x_t is stationary. [5 marks]

Q5. (a) The following table shows the population (in millions) of a certain country:

Census Year	1929	1939	1949	1959	1969	1979	1989
Population	15	18	17	20	24	23	21

Fit an exponential trend $x_t = ab^t$ to the above data where

$t = \frac{Census\ year - 1959}{10}$ by the method of least square and find the trend values.

Estimate the population in 1999 and 2001. [10 marks]

(b) Show that the autocorrelation function of the q-th order moving average process given by

$$Xt = \sum_{t=0}^{q} \frac{e_t - k}{m+1} \quad \text{is} \quad \rho(h) = \begin{cases} 1 & , \quad h = 0 \\ \dfrac{q+1-h}{q+1} & , \quad h = 1, 2, \ldots m \\ 0 & , \quad \text{elsewhere} \end{cases}$$ [10 marks]

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